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Chapter 4

RISK AND RETURN

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The value of investment is determined by risk and return, i.e., value of an investment is a function of the expected size and riskiness of return from it. Investors prefer larger returns to smaller returns, hence risk remaining the same, larger the expected return higher the investment value and vice-versa. They dislike risk. This dislike for risk is termed as risk-aversion. The degree of risk-aversion differs among investors and from time to time. The relation between degree of risk-aversion and investment value is negative, i.e. as the degree of risk-aversion increases, the value of investment decreases and vice versa.

Return comprises the income, which is in the form of dividends or interest, and the capital gain (loss). Return is calculated with the help of wealth ratio

$$\text{Wealth Ratio} = \frac{[\text{Income from the investment during a period + value of the investment at the end of the period}]}{\text{Net Amount Invested in the beginning of period.}}$$

Q. No.1(a): On 1.1.2003, Madhavji purchased an equity share of Mathura Ltd. at a Cum Dividend price of Rs.107 per share. The company paid a dividend of Rs.5 Per share for the year 2002. In October, 2003, the company paid an interim dividend of Rs.10 per share . On 31^{st} December, 2003, the market price per share was Rs.120 (cum – dividend price including final dividend of Rs5 per share). Find the return on investment for the year 2003.

Answer :

$$\text{Wealth ratio} : \frac{10 + 120}{107 - 5} = 1.2745$$

Return = 0.2745 i.e. 27.45%

Q.No.1 (b): On 1.1.2004, Keshavji purchased an equity share of Dwarka Ltd at a cum-dividend price of Rs.120 per share. The paid a dividend of Rs.5 per share for the year 2003. The company paid an interim dividend of Rs.5 per share in August, 2004 and another interim dividend of Rs.4per share on 10th October, 2004. On 31^{st} December, 2004, the market price per share was Rs.140 (cum – dividend price including final dividend of Rs6 per share). Find the return on investment for the year 2004.

Answer

5 + 4 + 140

Wealth ratio : -------------- = 1.2957
120 –5

Return = 0.2957 i.e. 29.57%

Q. No.1 (c): Continuing with example (b), the company paid an interim dividend of Rs.6 per share in August, 2005 and another interim dividend of Rs.5 per share on 10th November, 2005. On 31st December, 2005, the market price per share was Rs.140 (cum – dividend price including final dividend of Rs7 per share). Find the return on investment for year 2005.

Answer :

$$\text{Wealth ratio} : \frac{6 + 5 + 140}{140 - 6} = 1.1269$$

Return = 0.1269 i.e. 12.69 %

Risk refers to the possibility that the expected return may not materialize. There may be loss of capital, i.e. investment has to be sold for an amount less than paid for it. There may be no income from investment or the income may be less than the expected. The natural query is "Why the investors go for risky investment"? The answer is that the desire for higher return entices them to go for risky investments.

Investment decision should be taken after considering both return and risk. How to measure the risk? Standard deviation of various possible rates of return is used to measure the risk? Larger the standard deviation, greater the risk, and vice versa.

How to take investment decisions when various opportunities are there? Here two sets of 3 total rules provide help to us. These rules are:

SET A :

(i) If expected returns from various securities are different but their standard deviations are same: Decision should be taken on the basis of expected returns. Security with higher expected return is preferred.

(ii) If expected returns from various securities are same but their standard deviations are different: Decision should be taken on the basis of standard deviations. Security with lower standard deviation should be preferred.

SET B :

(iii) If expected returns as well as standard deviations from various securities are different, decision should be taken on the basis of coefficient of variation. Coefficient of variation is obtained by dividing standard deviation by expected return. Coefficient of variation defines risk as standard deviation per rupee of expected return. Security with lower coefficient of variation is preferred.

<u>Example:</u> Following is the data regarding six securities:

	A	B	C	D	E	F
Returns (%)	8	8	12	4	9	8
Risk (Standard Deviation)	4	5	12	4	5	6

Which of the securities will be selected?

Answer

(i) A, B and F have same return. A's SD is lowest. Hence, B and F should be dropped. Now we are left with A, C, D and E.

(ii) A and D have same SD. D's return is lower. Hence D should be dropped. Now we are left with A, C and E.

(iii)

	Return	SD	Coefficient of variation
A	8	4	0.50
C	12	12	1.00
E	9	5	0.56

The following securities may be selected in the order of:
(i) A (ii) E and (iii) C

<u>CALCULATION OF MEAN RETURN:</u>

Calculation of mean return can be explained with the help of following two examples:

(a) Find the mean return of the shares of particular company over 5 years:

Year	2000	2001	2002	2003	2004
Return %	16	6	-5	30	42

Answer: Let the return of the share = X

Year	X
2000	16
2001	6
2002	-5
2003	30
2004	42
	$\sum X = 89$

Mean = $\sum X/n$ = 89/ 5 = 17.80

(b) Find the mean return of the shares of a company:

Return %	16	6	-5	30	42
Probability	0.10	0.20	0.40	0.20	0.10

Teaching note: (the mean return calculated in this case, i.e. when probabilities are given, is also known as expected return. In fact, expected return, calculated on the basis of probabilities, is weighted average mean return, weights being probabilities.)

Answer: Let the return of the share = X

X	Probability (p)	pX
16	0.10	1.60
6	0.20	1.20
-5	0.40	-2.0
30	0.20	6.00
42	0.10	4.20
	$\sum p = 1$	$\sum pX = 11$

Mean return = $\sum pX / \sum p$ = 11 / 1 = 11

<u>CALCULATION OF STANDARD DEVIATION</u>

SD measures the variation in the values of the variable. In financial management, it is used as the measurement of the risk. The absolute values of the SDs do not convey any meaning. (For example, if the SD of returns of a particularly investment over 5 year is 20, it do not convey any meaning). If we are given SDs of two or more investments; from their comparison we can rank them of the basis of the risk involved. (Suppose, there are three investment opportunities- A, B and C with SDs being 10, 15 and 12 respectively. From this information, we can conclude that B has maximum risk, A has minimum risk; C's risk is more than that of A and less than that of B).

Variance is also a measurement of risk. *Variance is* $(SD)^2$. The absolute values of the Variances do not convey any meaning. When used for comparison purpose, variances give the same result as is given by SDs.

(Suppose, there are three investment opportunities- A, B and C with SDs being 10, 15 and 12 respectively. From this information, we can conclude that B has maximum risk, A has minimum risk; C's risk is more than that of A and less than

that of B. If we calculate the variances for A, B and C, the values would be 100, 225 and 144 respectively. If rank the three investments on the basis of risk, our conclusion is same and that is : A has minimum risk; B has maximum risk, and C's risk is more than that of A and less than that of B).

Calculation of Standard deviations can be explained with the help of following two examples:

Example (a) Find the SD of the rate of returns on the shares of particular company over five years :

Year	2000	2001	2002	2003	2004
Rate of return (%)	10	20	-5	12	13

Answer

Let the rate of return (%) = X

	X	x	x^2
2000	10	0	0
2001	20	10	100
2002	-5	-15	225
2003	12	2	4
2004	13	3	9
	$\sum X = 50$		338

Mean = $\sum X / n = 50 / 5 = 10$

SD = $\sqrt{(\sum x^2/n)} = \sqrt{(338/5)} = \sqrt{(67.20)} = 8.22$

Example (b) Find the SD of the rate of returns on the shares of particular co.:

Rate of Return(%)	10	20	30	20	10
Prob.	0.10	0.20	0.40	0.20	0.10

Answer: Let the rate of return (%) = X

X	P	Px	x	x^2	px^2
10	.10	1	-12	144	14.40
20	.20	4	-2	4	0.80
30	.40	12	8	64	25.6
20	.20	4	-2	4	0.80
10	.10	1	-12	144	14.40
		$\sum pX = 22$		$\sum x^2 = 360$	$\sum px^2 = 56$

Mean = $\sum pX / \sum p = 22/1 = 22$ SD = $\sqrt{(\sum px^2/\sum p)} = \sqrt{(56/1)} = 7.48$

COEFFICIENT OF VARIATION

= (SD/mean) x 100. It refers to the risk per rupee of return. For example, if the coefficient of variation is 20%, it means for earning an income of rupee one, the investor has to take the risk of loss of Re.0.20. Moderate investors[1] take decisions on the basis of coefficient of variation. Lower coefficient of variation is preferred by such investors.

Q. No.2: Shares A and B have the following probability distributions of possible future returns.

Probability	A(%)	B (%)
0.1	16	-20
0.2	06	10
0.4	-5	20
0.2	30	30
0.1	42	50

Calculate the expected rate of return for each share and standard deviation for each share. Calculate coefficient of variation for each share. Which share would you prefer?

Answer

(Company A)

Let return is denoted by X:

X	p	pX	x	px^2
16	0.10	1.60	5	2.50
06	0.20	1.20	-5	5.00
-5	0.40	-2.0	-16	102.40
30	0.20	6.00	19	72.20
42	0.10	4.20	31	96.10
	$\Sigma p = 1$	$\Sigma pX = 11$		$\Sigma px^2 = 278.20$

Mean = $\Sigma pX / \Sigma p = 11/1 = 11$ SD = $\sqrt{(\Sigma px^2/\Sigma p)} = \sqrt{(278.20/1)} = 16.68$

Coefficient of variation = 16.68/11 = 1.52

Similar calculations for B reveals: (Mean 19; S.D. 17; C. of V 0.89.)

Share B may be preferred because of lower amount of Coefficient of variation.

Q. No. 3: Following information is available in respect of dividend, Market price and market condition after one year:

Market condition Probability Market price Dividend per share

[1] Moderate investors take the investment decisions after considering both risk and return. Such investors neither go for wild investment opportunities (i.e. investment opportunities which are likely to offer very high returns and also involving very high risk) nor do they go for investment opportunities with very low risk witch offer very low return.

Good	0.25	115	9
Normal	0.50	107	5
Bad	0.25	97	3

The existing market price of an equity share is Rs.106 (FV Re.1) which is cum 10% bonus debenture of Rs.6 per share. M/s X Finance Company Ltd has offered the buy back of debenture at face value. Find out the expected return and variability of returns of the equity shares. And also advise : whether to accept buy back offer? **(NOV. 2005)**

Answer

Market condition	Wealth ratio	r % (X)	p	pX	px^2
Good	(115+ 9)/100	24	.25	6	36
Normal	(107+ 5)/100	12	.50	6	0
Bad	(97+ 3)/100	0	.25	0	36
				12	72

Expected Return = 12 Variability of returns i.e. SD = $\sqrt{(72)}$ = 8.49

If *the coupon rate of the debenture exceeds current market interest rate*, MV of the debenture will be more than the face value. In this scenario, the buy back offer should not accepted. Either the investor may hold the debenture and earn interest at a rate higher than the market, or he may sell in the open market where he/she will get more value than the face value.

If the coupon rate of the debenture is less than the current market interest rate, MV of the debenture will be lower than the face value. In this scenario, the buy back offer should be accepted.

SYSTEMATIC AND UNSYSTEMATIC RISK

Systematic risk refers to variability in return on investment due to market factors that affect all investments in a similar fashion. Examples of such factors are: Level of economic activities (recession or boom), variation in interest rates, inflation, political developments, etc. Unsystematic risk arises from such factors which are concerned with the firm. This risk is unique to a particular security. Examples are: strike, change in management, special export order, etc. (Unsystematic risk is also referred as firm-specific risk, it is denoted by ei) Unsystematic risk is called as diversifiable risk as it can be reduced with the help of diversification, i.e. instead of investing in the shares of one company, one may invest in the shares of various companies. Systematic risk is non-diversifiable; it cannot be reduced through diversification. All equity investors have to bear this risk.

The total risk, both systematic and unsystematic risk, of a security or portfolio is measured by the standard deviation.

Teaching note : Market portfolio (a portfolio of all the securities quoted in the stock exchange) has to bear only systematic risk. Market portfolio is a very well diversified portfolio. The unsystematic risk of the investment in the market portfolio is eliminated through diversification.

- Total risk of market portfolio = Systematic risk of market portfolio
- Unsystematic risk of market portfolio = 0

BETA

Beta is an indicator of an investment's systematic risk. It represents systematic risk associated with an investment in relation to total risk associated with market portfolio. (If the Beta of a security is 1.50, it does not mean that the systematic risk of the security is 1.50; it simply means that the security is 1.50 times riskier as compared to the market as a whole). Suppose the beta value of a particular security is 1.20, it means that if return of market portfolio varies by one per cent, the return from that security is likely to vary by 1.20 per cent. Therefore, this security is riskier than the market because we expect its return to fluctuate more than the market on a percentage basis. This beta measures the riskiness of individual security relative to market portfolio. It is a ratio of "its covariance with the market" to "the variance of market as a whole". A security with beta greater than one is called as aggressive security; with beta less than one is called as defensive security and with beta equal to one is called as neutral security.

$$\text{Beta} = \frac{\text{Covariance between returns from market Portfolio and those from particular security}}{\text{Variance of market portfolio}}$$

Beta of market portfolio is taken as 1.

Covariance : It is a statistical measurement that measures the combined variation (co-vary) between two variables; (that is, more or less when one of them is above its mean value, then the other variable tends to be above its mean value too, then the covariance between the two variables will be positive. On the other hand, if one of them is above its mean value and the other variable tends to be *below* its mean value, then the covariance between the two variables will be negative). In the Financial Management, it is used to measure the co-movements between return from 'market' and that from a particular security or portfolio. The range of covariance values is unrestricted (unlike the coefficient of correlation which is restricted to ± 1.)

Covariance = ∑xy/ n Where x is X – average value of X; y = Y- average value of Y.

Example

(X = return on market portfolio; Y = return on specific security)

Year	X	Y	x	y	xy	x^2
1981	15	16	0	0	0	0
1982	14	12	-1	-4	4	1
1983	17	19	2	+3	6	4
1984	16	18	1	+2	2	1
1985	13	15	-2	-1	2	4
	75	80	0	0	14	10

$\overline{X}=15$ $\overline{Y}=16$; $x = X - \overline{X}$, $y = Y - \overline{Y}$

$$\text{Covariance between X \& Y} = \frac{\sum xy}{n} = \frac{14}{5} = 2.80$$

$$\text{Variance of market portfolio} = \frac{\sum x^2}{n} = \frac{10}{5} = 2$$

$$\text{Beta} = \frac{2.80}{2} = 1.40.$$

Teaching note :

- ❖ Standard Deviation measures the total risk (Systematic as well as unsystematic risk)of an investment.
- ❖ Beta represents systematic risk associated with an investment in relation to total risk associated with market portfolio.
- ❖ Covariance is the measure of how much two variables vary together. That is to say, the covariance becomes more positive for each pair of values which differ from their mean in the same direction, and becomes more negative with each pair of values which differ from their mean in opposite directions. In this way, the more often they differ in the same direction, the more positive the covariance, and the more often they differ in opposite directions, the more negative the covariance.

  Covariance is a measure of the co-movement between two random variables.

A negative covariance means that variables move in different directions. A positive covariance means they move in the same direction.

Covariance can range from negative infinity to positive infinity. Covariance is an absolute measure. Covariances cannot be compared with one another.

- Coefficient of correlation = [Covariance) / (SDx.SDy)]

Coefficient of Correlation (r) states relationship between two variables; the two variables may be return from two securities, or return from market portfolio and return from a security, rainfall and agriculture output, inflation and interest rate etc. r is always between −1 to + 1.

If r = + 1, it means that both the series are moving I n the same direction and with the same percentage. For example, if one increases by 10%, the other also increases by 10%.
If decreases by 10%, the other also decreases by 10%; and so on.

If r = −1, it means both the series move with same percentage but in the reverse direction. For example, if one increase by 10%, the other decreases by 10%.

The other positive values of r indicate that more or less both the series move in the same direction (if one increased, the other also increases; if one decreases the other also decreases) but the rates of changes are different. For example, if increases by 5%, the other may increase by 2%.

The other negative values of r indicate that more or less both the series move in the reverse direction (if one increases, the other decreases; if one decreases the other increases) and the rates of changes are different. For example, if increases by 5%, the other may decrease by 2%.

Q..No.4. Using the following data regarding two securities C and D, find which of the two securities is more risky? Why?

	C	D
Average return	15%	18%
Standard deviation of returns of past	0.20	0.15
Correlation coefficient with market	0.50	0.80
Beta	0.65	0.78

Find market portfolio variance.

Answer: Security C is more risky as its SD (which is a measurement of total risk) is greater than that of D

Correlation = [(covariance)]/ [(market SD) (0.20)]
0.50 =[(covariance)]/ [(market SD) (0.20)]
0.10 = covariance / market SD …………(1)
Beta = covariance / (Market SD)^2
0.65 = covariance / (Market SD)^2 ………(2)

Solving the two equations, Market SD = 0.1538
Market variance :0.023669

TOTAL RISK OF AN INVESTMENT

Total risk of an investment, which is variance (or standard deviation) of its return, can be divided into two parts:
Total risk = Systematic risk + unsystematic risk

Systematic risk can be measured with the help of Beta (Beta indicates the riskiness of an investment, it relation to market portfolio.)

(i) Systematic Risk of an investment =
Beta of that investment x Market Standard Deviation

Suppose the market SD is 5. It means the total risk of the market portfolio is 5 (Remember that total risk of the market portfolio is only systematic risk. Market portfolio is a very well diversified portfolio. The unsystematic risk of the investment in the market portfolio is eliminated through diversification.). Suppose there is security having Beta of 1.20. It means systematic risk of the security is 1.20 times the systematic risk of the market portfolio; in other words, the systematic risk of the security is 6.

(ii) Unsystematic Risk of an investment =
Total risk (SD of that investment)-Systematic Risk(calculated as above)

ALTERNATIVE APPROACH:
(i) Systematic Risk of an Investment =
Beta^2 x Market Variance
(ii) Unsystematic Risk of an investment =
Total risk (Variance of the investment)-Systematic Risk(calculated above)

Unsystematic risk is also referred as Residual risk, also firm specific risk, also risk not related to market Index. It is denoted by ei.

Q. No.5: The following are the estimates for two stocks:

Stock	Expected Return	Beta	Residual SD
A	13%	0.80	30%
B	18%	1.20	40%

Market SD is 20%. What are the SDs of A and B?

Answer

Systematic risk of A: Beta of A x Market Standard Deviation
= 0.80 x 20% = 16%

Total Risk of A (SD of A) = 16 + 30 = 46%

Systematic risk of B: Beta of B x Market Standard Deviation
= 1.20 x 20% = 24.00%

Total Risk of B (SD of B) = 24+ 40 = 64%

CAPITAL ASSET PRICING MODEL

CAPM explains the required return (i.e. the minimum rate of return which induces the investors to select a particular investment) in the form of the following equation:

K = RF + RP

K = Required rate of return

RF = Risk free rate of return

RP = Risk premium

Risk premium is additional return expected by the investor for bearing the additional risk associated with a particular investment. It is calculated as Beta X (RM-RF) where RM is expected return on market portfolio.

Suppose beta of a security is 1.21

RF = 7 per cent, RM = 13 per cent

K = 7 + 1.21 (13 - 7) = 14.26 per cent

Investor will require a return of 14.26 per cent return from this investment. He can get 7 per cent return without taking any risk. Market portfolio offers him extra 6 per cent return where risk is lesser as compared to risk from this security. Risk from this security is 1.21 times as compared to risk from market portfolio. Hence premium is 6 x 1.21 = 7.26 per cent. Thus required rate of return is equal to risk free return + risk premium.

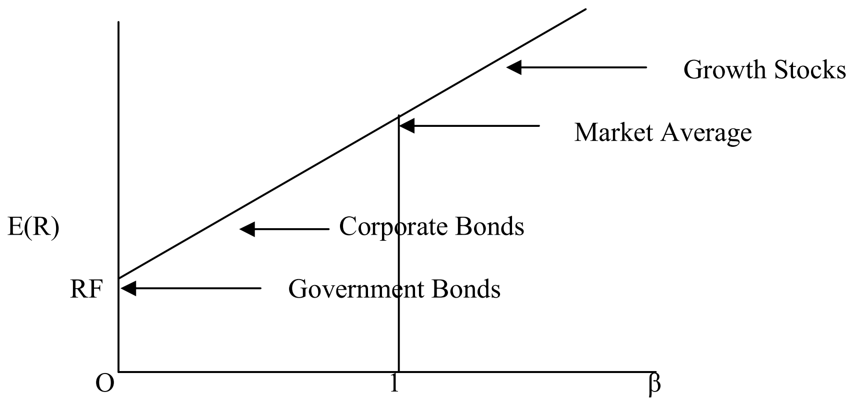

DIAGRAM 1

Q. No. 6 : Beta 1.08, RF 10 per cent, RM 15 per cent, dividend per share expected at the year-end Rs.2.00. Dividend is likely to grow at 11 per cent p.a. for years to come. Market price of share?

Answer:

Ke = RF + β(RM-RF) = 10 + 1.08(15-10) = 15.40

$$P = \frac{D_1}{Ke - g} = \frac{2}{0.1540 - 0.11} = 45.45$$

Q. No 7: Covariance of returns between market and equity shares of XYZ Ltd is 10%. Market SD is 40%. RM = 20%. RF is 12%. Calculate Ke of XYZ Ltd.

Answer:

SD = 0.40 Variance=0.40 x 0.40=0.16 Variance (%) = 0.16 x 100 = 16

$$\text{Beta} = \frac{\text{Covariance}}{\text{Market variance}} = \frac{10\%}{16\%} = 0.625$$

ALTERNATIVE WAY:

$$\text{Beta} = \frac{\text{Covariance}}{\text{Market variance}} = \frac{0.10}{(0.40)^2} = 0.625$$

Ke = RF + Beta(RM-RF) = 12 + .625(20-12) = 17

Q. No. 8 : Security S.D.= 3% Market S.D. = 2.20%

Coefficient of correlation for security with market = 0.80

Return from market portfolio= 9.80%. Risk Free rate of return = 5.20%

Find the required return from the security. **(May, 1998)**

Answer:

$$\text{Coefficient of correlation} = \frac{\text{Covariance}}{(\text{SDsecuirty}).(\text{SDmarket})}$$

$$0.80 = \frac{\text{Covariance}}{(0.03).(0.022)}$$

Covariance = 0.000528

Beta = Covariance /(Market variance)

$$= 0.000528 / (0.0220)^2$$

$$= 1.091$$

Required return from the security = RF + Beta (RM-RF)

$$= 5.20 + 1.091(9.80 - 5.20) = 10.22\%$$

Q. No. 9: The market price of the equity share of Nandnandan Ltd is Rs.50. Ke = 14%. RF = 5%. Risk premium of market portfolio = 10%. It is expected that the company shall be paying constant dividend year after year. What shall be the market price of share if, r between the return from this security and that from market portfolio is halved (the values of SDs remain unchanged)?

Answer

14 = 5 + Beta (10)
Beta = 0.90
Ke = Dividend per share / P
0.14 = D / 50
D = 7

If r is halved, Beta would be equal to 0.45.
Ke = 5 + 0.45(10) = 9.50%
Ke = D / P
0.095 = 7 /P P = 73.68

Q. No. 10: The expected rate of return on market portfolio is 20%. The Beta of a security is 1.00. Dividend yield (Dividend per share / market price per share) is 5%. What is the expected rate of the price appreciation on price of that security?

Answer

Total required rate of return from the security = 20%
Dividend yield = 5%
Price appreciation = 15%

Overall Beta

The discussion contained in the above paragraph relates to a particular security. Beta may also be calculated for the firm as a whole. This Beta is referred as Firm Beta or Overall Beta or Assets Beta. Overall Beta indicates expected change in return from the firm as a whole when the return from market portfolio varies by 1 percent. Overall Beta is weighted average of Equity Beta & Debt Beta. (If debt Beta is not given in question, it is assumed to be zero).

Overall Beta (If Tax Ignored) : $$\text{Debt Beta} \times \frac{D}{D+E} + \text{Equity Beta} \times \frac{E}{D+E}$$

Overall Beta (Tax considered):

D(1-T) E

$$= \text{Debt Beta} \times \frac{}{D(1-T)+E} + \text{Equity Beta} \times \frac{}{D(1-T)+E}$$

A school of thought led by MM believe that overall Beta is not affected by change in Capital structure.

Q. No. 11 : The capital structure of Madhav Ltd is as follows :

	Beta	Amount Rs. Million
Debt	0	150
Preference shares	0.20	50
Equity shares	1.20	200

Find the beta for the overall beta of the company. How the overall beta change if the company raises Rs.200m by issuing new equity shares and use this amount for redeeming the debt and Preference shares?

Answer

W_1 = 150/400 = 0.375 W_2 = 50/400 = 0.125 W_3 =200/400=0.50

Overall Beta = (0)(0.375) + (0.20)(0.125) + (1.20)(0.50) = 0.625

According to MM, the change in the capital structure does not change the overall beat. Hence, the company action will have no effect on the overall beat i.e. the overall beat will remain unchanged.

Q. No. 12 :

A Company's capital structure comprises equity share capital having market value of Rs.80 crores plus Rs.50 crores debentures. The debt beta coefficient may be assumed to be 0.25. The current risk – free rate is 8% and the market rate of return is 16%. Equity Beta = 1.40, Find Ko. Ignore Tax.

Answer

$$\text{Overall } \beta = D.\beta \times \frac{D}{D+E} + E.\beta \times \frac{E}{D+E}$$

Overall β = [(0.25) X (50) / (50 + 80)] + [(1.40)X(80)/(50+ 80)] = 0.9577

Ko[2] = RF + Overall β (RM-RF)

= 8 + .9577(16-8) = 15.66%

Alternative way of calculation of Ko :

Kd = RF + Debt β (RM-RF)

= 8 + 0.25 (16 -8) = 10%

Ke = RF + Equity β (RM-RF)

[2] This method of calculating Ko may be applied only when tax is ignored. If tax is to be considered, only alternative method given in this answer can be applied.

$= 8 + 1.40\ (16-8) = 19.20\%$

X	W	XW
10.00	50	500
19.20	80	1,536
	130	2,036

Ko = 2036/130 = 15.66%

Q. No. 13 :

The total market value of the equity share of O.R.E Company Rs.60,00,000 and the total value of the debt is Rs.40,00,000. The treasurer estimate that the beta of the equity is currently 1.5 and that the expected risk premium on the market is 10 per cent. The Treasury bill rate is 8 per cent. Ignore Tax.

Required:

(1) What is overall Beta?

(2) Estimate Ko.

Answer

$$\text{Overall } \beta = D.\beta \times \frac{D}{D+E} + E.\beta \times \frac{E}{D+E}$$

$$\text{Overall } \beta = \left(0 \times \frac{4}{4+6}\right) + \left(1.50 \times \frac{6}{4+6}\right) = .90$$

[3]Ko = RF + Overall β (RM−RF)

$= 8 + .90(10) = 17\%$

Alternative way of calculation of Ko :

Kd = RF + Debt β (RM−RF)

$= 8 + 0\ (10) = 8\%$

Ke = RF + Equity β (RM−RF)

$= 8 + 1.50\ (10) = 23\%$

X	W	XW
8	40	320
23	60	1380
	100	1,700

Ko = 1,700/100 = 17%

[3] This method of calculating Ko may be applied only when tax is ignored. If tax is to be considered, only alternative method given in this answer can be applied.

Q. No. 14 :

A project had an equity beta of 1.2 and was going to be financed by a combination of 30% debt and 70% equity. Assuming debt-beta to be zero, calculate the Project beta taking risk-free-rate of return to be 10% and return on market portfolio at 18%. Ignore Tax. Ko? **(May, 2002)**

Answer

$$\text{Overall } \beta = D.\beta \times \frac{D}{D+E} + E.\beta \times \frac{E}{D+E}$$

$$\text{Overall } \beta = \left(0 \times \frac{30}{30+70}\right) + \left(1.20 \times \frac{70}{30+70}\right) = 0.84$$

[4]Ko = RF + Overall β (RM-RF)
= 10 + .84(18-10) = 16.72

Alternative way of calculation of Ko:

Kd = RF + Debt β (RM-RF)
= 10 + 0 (18-10) = 10%

Ke = RF + Equity β (RM-RF)
= 10 + 1.20 (18-10) = 19.60%

X	W	XW
10.00	30	300
19.60	70	1,372
	100	1,672

Ko = 1,672/100 = 16.72%

Q. No. 15 :

Given Equity Beta 0.90, Debt Beta 0. Tax NIL. Debt: Equity .50 / .50. What will be new equity Beta if debt / equity is changed to 0.30 / .70 by issuing additional equity at Market price to redeem 40% of existing Debt?
What will be your answer if tax rate is 40%.

Answer

NO TAX

[4] This method of calculating Ko may be applied only when tax is ignored. If tax is to be considered, only alternative method given in this answer can be applied.

Firm Beta before redemption of debt

$$= 0 \times \frac{0.50}{.50+.50} + 0.90 \times \frac{0.50}{.50+.50}$$

$= .45$

After redemption of debt by issuing equity shares,

$$0.45 = 0 \times \frac{0.30}{.30+.70} + E.\beta \times \frac{0.70}{.30+.70}$$

Equity β = 0.64

TAX 40%

Firm Beta before redemption of debt =

$$= 0 \times \frac{0.50(1-0.40)}{.50+.50(1-.40)} + 0.90 \times \frac{0.50}{.50+.50(1-.40)}$$

$= .5625$

After redemption of debt by issuing equity shares,

$0.5625 =$

$$0 \times \frac{0.30(1-0.40)}{.70+.30(1-.40)} + E.\beta \times \frac{0.70}{.70+.30(1-.40)}$$

Equity β = 0.7071

Q. No. 16 :

A Company's capital structure comprises equity share capital having market value of Rs.80 crores plus Rs.50 crores debentures. The debt beta coefficient may be assumed to be 0.25. The current risk – free rate is 8% and the market rate of return is 16%. Equity Beta = 1.40, Find Overall Beta. Find Ko. Tax30%

Answer

Overall Beta =

$$= 0.25 \times \frac{50(1-0.30)}{80+50(1-.30)} + 1.40 \times \frac{80}{80+50(1-.30)} = 1.05$$

Calculation of Ko :

Kd = [RF + Debt β (RM-RF)] x [1-tax rate]

= [8 + 0.25 (16 -8)][1-0.30] = 7%

Ke = RF + Equity β (RM-RF)

= 8 + 1.40 (16-8) = 19.20%		
X	W	XW
7.00	50	350
19.20	80	1,536
	130	1,886

Ko = 1,886/130 = 14.51%

Q. No. 17 :

A Ltd' s equity Beta is 1.25. Its capital structure is 30% debt and 70% equity. B Ltd is an identical company except that its gear is 40% debt and 60% equity. Tax rate is 30%. Find equity Beta of B Ltd.

Answer

OVERALL BETA (A) =

$$0 \times \frac{30(1-\text{Tax rate})}{70 + 30(1-\text{tax rate})} + (1.25) \times \frac{70}{70 + 30(1-\text{tax rate})} = 0.9615$$

OVERALL BETA (B) =

Finance 40 % debt & 60 % equity

$$0.9615 = 0 \times \frac{40(1-.30)}{60 + 40(1-.30)} + \text{E. Beta} \times \frac{60}{60 + 40\,(1-0.30)}$$

Solving above equation, E. β = 1,4102.

Q. No. 18: A Furniture Ltd is planning to form a subsidiary company which will be dealing in Fabrics. Current equity Beta of A Furniture Ltd is 1.70. The fabrics industry's current equity Beta is 1.60 . The fabrics industry has 30% debt and 70% equity. With RM = 25% , RF = 10%, tax = 30% and debt Beta =0, find the overall cost of capital. How your answer change if gearing is 50% and 50%? What if the project is wholly equity financed?

Answer

Overall cost of capital of fabrics with **30% debt and 70% equity**

Ke = 10 + 1.60 (25 −10) = 34

Kd = 10 (1− 0.30) = 7

Source	Cost (X)	W	XW
EQUITY	34	0.70	23.80
DEBT	7	0.30	2.10

			25.90

Ko = 25.90 %

Finance 50 % debt & 50 % equity

Overall Beta (Fabric Sector) =

$$0 \times \frac{30(1-\text{Tax rate})}{70 + 30(1-\text{tax rate})} + (1.60) \times \frac{70}{70 + 30(1-\text{tax rate})} = 1.231$$

$$1.231 = 0 \times \frac{50(1-.30)}{50 + 50(1-.30)} + \text{E. Beta} \times \frac{50}{50 + 50(1-0.30)}$$

Solving above equation, E. β = 2.09

Ke = 10 + 2.09(25-10) = 41.35

Source	Cost (X)	W	XW
EQUITY	41.35	0.50	20.675
DEBT	7	0.50	3.50
			24.175

Dis. Rate or overall cost of capital = 24.175/1 = 24.175%

Project financed by equity only:
Equity Beta = Overall Beta = 1.231
Ke = 10 + 1.231(25-10) =28.465 % Ko = 28.465%

PORTFOLIO THEORY

"Do not put all your eggs in the same basket". The wisdom of this maxim is that one should not put all his wealth in one asset only, rather one should invest in many assets. In other words, the maxim suggests diversification of investments for risk reduction.

Portfolio is a combination of securities. Combining securities in a portfolio can reduce the risk because some of the fluctuations offset each other. Investors can reduce risk by holding investments in diversified portfolio.

There are two theories of Portfolio Management, (a) Traditional Theory (b) Modern Theory. Both traditional as well as modern theories of the Portfolio Management find their foundations in the wisdom of the maxim.

The traditional theory does not suggest any methodology for making portfolio, the assets for constructing the portfolios are just to be picked up only on the basis of judgment.

Modern Portfolio Theory[5] provides a sound method for investors to establish a disciplined approach to investing. The Modern Portfolio theory (MPT) suggests a definite methodology[6] for this purpose. MPT is based on statistical methods (Mean, SD and coefficient of correlation). Using SD as a measurement of risk and coefficient of correlation for calculating portfolio risks are termed as major contributions of Markowitz, the father of MPT. The theory reveals that the degree of risk reduction depends upon correlation between returns from different investments. Lower the correlation between returns from securities, greater the risk reduction potential when the assets are combined to form a portfolio. If the correlation between returns from securities is + 1, their combination does not reduce the risk.

Teaching note – not to be given in the exam. We shall be studying, the relation between the value of coefficient of correlation between the returns from the securities and risk reduction potential of the portfolio constituting them, after studying the methods of calculating the portfolio risk.

There are two methods of calculating the return and risk of the portfolio (a) Direct method (b) Indirect method. (Risk may be calculated either Portfolio SD or portfolio variance.)

DIRECT METHOD

Under direct method, we calculate periodical returns of the portfolio. Mean of these returns represents portfolio *return* and *SD* of these returns represents *portfolio risk* (portfolio SD).

INDIRECT METHOD

Portfolio Return

The expected return on a portfolio of securities is the weighted average of the expected returns of the individual securities making up the portfolio. The weights are equal to proportion of the investment in each security in the portfolio.

---

[5] MPT was introduced by Markowitz in his paper "Portfolio Selection" which appeared in the Journal Finance (USA) in 1952 (At that time , he was a PhD student of Chicago School of Economics). Sharpe (a student of Markowitz) also contributed a lot towards further advancement of the theory. (Sharpe's main contribution was the development of Capital Assets Pricing Model). Both Markowitz and Sharpe shared Noble prize in Economics with Miller in 1990.

[6] We shall be studying this methodology (in brief) after studying the basics of the Portfolio Management.

Portfolio Risk

The risk a portfolio is measured by its variance or standard deviation (SD) of a Portfolio.

Variance of portfolio

$= W_1^2.(SD_1)^2 + W_2^2.(SD_2)^2 + 2.W_1.W_2.r_{12}..(SD_1).(SD_2)$

W_1	=	Proportion of investment in security A'.
W_2	=	Proportion of investment in security B'.
$(SD_1)^2$	=	Variance of returns from security A
$(SD_2)^2$	=	Variance of returns from security B
r_{12}	=	Coefficient of correlation between returns from securities A & B.

The above – mentioned formula is for calculating variance of a two-asset portfolio. Variance of "more than two – asset portfolio" can be calculated on similar lines.

For example, variance of three asset portfolio is:

$W_1^2.(SD_1)^2 + W_2^2.(SD_2)^2 + W_3{}^2.(SD_3)^2 + 2.r_{12}..W_1.W_2..SD_1.SD_2 + 2.r_{23}.. W_2.W_3.SD_2. SD_3 + 2.r_{13}.W_1.W_3.SD_1.SD_3$

Example

Year	Return from Security A	Return from Security B
2001	11	15
2002	13	9
2003	-8	27
2004	27	-3
2005	17	12

Suppose we invest 50% of funds in A and balance in B. Calculate the return and risk of the Portfolio.

DIRECT METHOD Let the return is denoted by X

Portfolio Mean = $\sum X/n = 60/5 = 12$

Portfolio Variance = $\sum x^2/n = 14.50/5 = 2.90$

Portfolio SD = $\sqrt{\sum x^2/n} = \sqrt{14.50/5} = 1.70$

INDIRECT METHOD

Let the return of A is denoted by X and that of B by Y.

X	x	x^2	Y	y	y^2	xy
11	-1	1	15	3	9	-3
13	1	1	9	-3	9	-3
-8	-20	400	27	15	225	-300
27	15	225	-3	-15	225	-225
17	5	25	12	0	0	0
$\sum X = 60$	$\sum x = 0$	$\sum x^2 = 652$	$\sum Y = 60$	$\sum$	$\sum y^2 = 468$	$\sum xy = -531$

Mean of X = 12   Mean of Y = 12

SD of X = $\sqrt{\sum x^2/n} = \sqrt{652/5}$ = 11.42

SD of Y = $\sqrt{\sum y^2/n} = \sqrt{468/5}$ = 9.68

Coefficient of correlation = ($\sum xy/n$) / (SDx.SDy)

(−531/5)/(11.42 . 9.68) = −0.9607

Portfolio Variance = $(W_1)^2.(SD_1)^2 + (W_2)^2.(SD_2)^2 + 2(W_1)(W_2)(r_{12})(SD_1)(SD_2)$

Portfolio SD = $\sqrt{(W_1)^2.(SD_1)^2 + (W_2)^2.(SD_2)^2 + 2(W_1)(W_2)(r_{12})(SD_1)(SD_2)}$

= $\sqrt{(0.50)^2.(11.42)^2 + (0.50)^2.(9.68)^2 + 2(0.50)(0.50)(-0.9607)(11.42)(9.68)}$

= 1.71

Q. No. 19 : Return from equity shares of two companies for last five years :

Year	Lalita Ltd.	Sakhi Ltd.
20x1	10%	20%
20x2	20%	10%
20x3	30%	-5%
20x4	-10%	15%
20x5	10%	20%

- An investor invests 50% of his investible funds in Lalita and balance in Sakhi. Find his expected return.
- Find SD of each stock
- Find covariance between Lalita Ltd and Sakhi Ltd.
- Find coefficient of correlation between the two.
- Find portfolio risk, by indirect method, if 40% in invested in the Lalita Ltd and balance in Salkhi Ltd.
- Find portfolio risk, by direct method, if 40% in invested in the Lalita Ltd and balance in Salkhi Ltd.

Answer

(a) Let return from Lalita Ltd. = X.   Let return from Sakhi Ltd. = Y

X	x	x^2	Y	y	y^2	xy
10	-2	4	20	8	64	-16
20	8	64	10	-2	4	-16
30	18	324	-5	-17	289	-306
-10	-22	484	15	3	9	-66

10	−2	4	20	8	64	−16
$\sum X=60$		$\sum x^2$ =880	$\sum Y$ =60		$\sum y^2$ =430	$\sum xy= -420$

Average return of Lalita = $\sum X/n$ = 60/5 =12

Average return of Sakhi = $\sum Y/n$ = 60/5 =12

Expected return of the portfolio = (.50)(12) + (.50)(12) = 12

(b) SD of shares of Lalita Ltd . $\sqrt{[\sum x^2/n]} = \sqrt{[880/5]}$ =13.27

SD of shares of Sakhi Ltd . $\sqrt{[\sum y^2/n]} = \sqrt{[430/5]}$ = 9.27

(c) covariance = $\sum xy$ / n = −420 / 5 = −84

(d) Coefficient of correlation=Covariance/[(SDx).(SDy)]
= −84 / [(13.27).(9.27)] = −0.68
(e) Portfolio SD =
$\sqrt{[(0.40)^2.(13.27)^2 + (0.60)^2.(9.27)^2 + 2(0.40)(0.60)(-0.68)(13.27)(9.27)]}$
= 4.34

(f) let the return from the portfolio = Z

Let return from portfolio = Z

Z	z	z^2
16	4	16
14	2	4
9	−3	9
5	−7	49
16	4	16
$\sum Z$ = 60		$\sum z^2 = 94$

Mean return from portfolio = $\sum Z/n$ = 60/5 =12

SD of portfolio = $\sqrt{\overline{[\sum pz^2/n]}}$ $\sqrt{[94 / 5]}$ = 4.34

Q. No. 20 : Calculate expected return and SD of each of following two investments P and Q . Also calculate the expected return and SD of a portfolio in which 50% of funds are invested in P and balance in Q. What if 40% invested in P and balance in Q?

State of Monsoon	Probability	Return from P	Return from Q
Poor	0.10	10	20
Below normal	0.20	20	30
Normal	0.40	30	40
Above normal	0,20	35	50
Excellent	0.10	40	70

Answer: Let return from P = X

X	p	pX	x	px^2
10	.10	1	-18	32.4
20	.20	4	-8	12.8
30	.40	12	2	1.6
35	.20	7	7	9.8
40	.10	4	12	14.4
		28		71

Expected return from P = ∑pX/∑p = 28/1 = 28

SD of P = $\sqrt{[(\sum px^2/\sum p)]}$ = = $\sqrt{[(71/1)]}$ = 8.43

Let return from Q = Y

Y	p	pY	y	py^2
20	.10	2	-21	44.1
30	.20	6	-11	24.2
40	.40	16	-1	0.40
50	.20	10	9	16.2
70	.10	7	29	84.1
		41		169

Expected return from Q = ∑pY/∑p = 41/1 =41

SD of Q = $\sqrt{[(\sum py^2/\sum p)]}$ = $\sqrt{[(169/1)]}$ = 13

Calculation of covariance and r

X	y	p	pxy
-18	-21	.10	37.8
-8	-11	.20	17.6
2	-1	.40	-0.8
7	9	.20	12.6
12	29	.10	34.8
			102

Covariance = ∑pxy / ∑p = 102/1 =102

Covariance 102

r = -------------- = ----------------- = 0.93
(SDx)(SDy) (8.43)(13)

50% in P and balance in Q:

Portfolio return = 28(.50) + 41 (.50) = 34.50

Portfolio SD= $\sqrt{[(0.50)^2.(8.43)^2 + (0.50)^2.(13)^2 + 2(0.50)(0.50)(0.93)(8.43)(13)]}$

= 10.53

40% in P and balance in Q:

Portfolio return = 28(.40) + 41 (.60) = 35.80.

Portfolio SD =$\sqrt{[(0.40)^2.(8.43)^2 + (0.60)^2.(13)^2 + 2(0.40)(0.60)(0.93)(8.43)(13)}$

= 11.01

Q.No.21: X Ltd is currently engaged in the business of making documentary films. The following information, relating to this company, is available:

Total investment in the business	:	Rs.10 Crores
Expected Return	:	20%
SD of returns	:	30%

The company is planning to go for the business of making feature films. The following information, relating to feature film business, is available:

Total investment in the business	:	Rs.30 Crores
Expected Return	:	40%
SD of returns	:	20%

Coefficient of correlation between returns from two businesses is 0.90.

X Ltd has a policy of evaluating new projects on the basis of following equation:

Net benefit from the project = 80 Return (%) – variance (%).

If the implementation of the project results in increase in the net benefit, the project is accepted. Should the project be accepted?

Answer

Expected return after new business = 20(.25) + 40(.75) = 35

Variance after new business =

$(0.25)^2.(0.30)^2 + (0.75)^2.(0.20)^2 + 2(0.25)(0.75)(0.90)(.30)(.20) = 0.048375$

= 4.8375 %

Variance before new business = $(0.30)^2$ = 0.09 = 9%

Net benefit before business = 80(20) – 9 = 1591

Net benefit after business = 80(35) – 4.8375 = 2795.1625

As the benefit after the new business is increased, the new business is recommended.

Q. No. 22 : X Co., Ltd., invested on 1.4.2005 in certain equity shares as below:

Name of Co.	No. Shares	Cost (Rs.)
M Ltd.	1,000 (Rs.100 each)	2,00,000
N.Ltd.	500 (Rs. 10 each)	1,50,000

In September, 2005, 10% dividend was paid out by M Ltd. and in October, 2005, 30% dividend paid out by N Ltd. On 31.3.2006 market quotations showed a value of Rs.220 and Rs. 290 per share for M Ltd. and N Ltd respectively.

On 1.4.2006, investment advisors indicate (a) that the dividends from M Ltd. and N Ltd. for the year ending 31.3.2007 are likely to be 20% and 35% respectively and (b) that the probabilities of market quotations on 31.3.2007 are as below:

Probability factor	Price/share of M Ltd.	Price/share of N Ltd.
0.2	220	290
0.5	250	310
0.3	280	330

You are required to:

(i) Calculate the average return from the portfolio for the year ended 31.3.2006;

(ii) Calculate the expected average return from the portfolio for the year 2006-07;

(iii) Advise X Co. Ltd., of the comparative risk in the two investments by calculating the standing deviating in each case. **(Nov. 2006) (May, 2008)**

Answer (i)

Year end wealth : Cash (received on account of dividend from M)= 10000

\+ Cash (received on account of dividend from N)= 1500

\+ Market value of shares of M = 2,20,000

\+ Market value of shares of N = 1,45,000 = 3,76,500

Investment in the beginning of the year= 2,00,000 + 1,50,000= 3,50,000

Average return from the portfolio for the year ended 31.3.2006 :

$$(3,76,500 / 3,50,000) - 1 = 0.0757 = 7.57\%$$

(ii) Expected share price of M = 220x.2 + 250x.5 + 280x.3 = 253

Expected share price of N = 312

Year end wealth : Cash (received on account of dividend from M)=20000

+ Cash (received on account of dividend from N)= 1750

+ Market value of shares of M = 2,53,000

+ Market value of shares of N = 1,56,000 = 4,30,750

Investment in the beginning of the year = 2,20,000 + 1,45,000 = 3,65,000

Average return from the portfolio for the year ended 31.3.2007 :

= (4,30,750 / 3,65,000) - 1

= .1801 i.e. 18.01 %

SD of M

Return per share (X)	p	pX	x	px^2
20	.2	4	−33	217.80
50	.5	25	−3	4.50
80	.3	24	+ 27	218.70
		53		441

SD of M = √441 = 21

SD of N

Return per share (X)	p	pX	x	px^2
03.50	.2	0.70	−22	96.80
23.50	.5	11.75	−2	2
43.50	.3	13.05	18	97.20
		25.50		196.00

Variance = 196 SD = 14

OBJECTIVE OF PORTFOLIO MANAGEMENT:

The fundamental objective of the portfolio management is risk reduction through diversification. This objective is said to have achieved if portfolio standard deviation is less than weighted average of standard deviations of securities constituting the portfolio, weights being proportion of investment in each security in the portfolio.

The degree of risk reduction depends up on coefficient of Correlation. Lesser the coefficient of correlation, greater the risk reduction potential of the portfolio.

Maximum reduction is when r = −1. There is no risk reduction if r = + 1 (We shall be understanding this concept with the help of next question)

Portfolio objective :

<table>
<tr><td>
Portfolio SD < weighted average SD

Weighted average SD = weighted average of standard deviations of securities constituting the portfolio, weights being proportion of investment in each security in the portfolio.

Let SD of A =5 SD of B=6 proportion of weights= equal. r = −1.

Weighted average SD = 5x0.50 + 6x0.50 = 5.50

Portfolio SD = $\sqrt{(0.50)^2.(5)^2 + (0.50)^2.\ (6)^2 + 2(0.50)(0.50)(-1)(5)(6)}$ = 0.50

The portfolio has achieved its object of risk reduction. The securities constituting the portfolio and the proportion in which the investment has been made should have resulted in the risk of 5.50 while the portfolio SD is only 0.50.

Gain of portfolio = Weighted Av. SD – Portfolio SD
= 5.50 – 0.50 = 5
</td></tr>
<tr><td>
Gain of portfolio (%) =
$= \frac{\text{Weighted Av. SD} - \text{Portfolio SD}}{\text{Weighted average SD}} \times 100$
$= \frac{5.50 - 0.50}{5.50} \times 100 = 90.91\ \%$
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Q. No. 23 : S.D. of A = 5 S.D. of B = 6 Weights 0.4 : 0.6

Weighted average of SDs. = 5.60

Find portfolio S.D. if r = + 1; r =+ 0.5, r = 0; r = −0.5 and r = −1

Find Gain of Portfolio (%) under various values of r.

Answer

Weighted average SD = 5x0.40 + 6 x 0.60 = 5.60

Portfolio SD if r = + 1

$= \sqrt{(0.40)^2.(5)^2 + (0.60)^2.\ (6)^2 + 2(0.40)(060)(+1)(5)(6)}$ = 5.60

Portfolio SD if r = + 0.50

$= \sqrt{(0.40)^2.(5)^2 + (0.60)^2.\ (6)^2 + 2(0.40)(060)(0.50)(5)(6)}$ = 4.92

Portfolio SD if r = o

$= \sqrt{(0.40)^2.(5)^2 + (0.60)^2.\ (6)^2 + 2(0.40)(060)(0)(5)(6)}$ = 4.12

Portfolio SD if r = −0.50

$$= \sqrt{(0.40)^2.(5)^2 + (0.60)^2.\ (6)^2 + \ 2(0.40)(060)(-0.50)(5)(6)} \qquad = 3.12$$

Portfolio SD if r = −1

$$= \sqrt{(0.40)^2.(5)^2 + (0.60)^2.\ (6)^2 + \ 2(0.40)(060)(-1)(5)(6)} \qquad = 1.60$$

Coefficient of Correlation	Weighted average SD	Portfolio SD	Gain of portfolio (%)
+ 1	5.60	5.60	0
+ 0.50	5.60	4.92	[(5.60−4.92)/5.60]x100=12.14
0	5.60	4.12	[(5.60−4.12)/5.60]x100=26.43
−0.50	5.60	3.12	[(5.60−3.12)/5.60]x100=44.29
−1	5.60	1.60	[(5.60−1.60)/5.60]x100=71.43

The table illustrates that lower the coefficient of correlation, greater the risk reduction potential. Maximum risk reduction is when r = −1. There is no risk reduction when r = + 1.

Q. No. 24 : Vidurbhai is interested in investing in 2 out of following three shares. He wants to invest equal amounts in the shares suggested by you. You are given the following Variance-covariance Table. Suggest.

	Equity shares of Girdhari Ltd.	Equity shares of Banwari Ltd.	Equity shares of Murari Ltd.
Girdhari Ltd	16	0.90	0.70
Banwari Ltd	0.90	4	0.20
Murari Ltd.	0.70	0.20	16

Answer

Portfolio (Girdhari and Banwari) Variance :

$= (0.50)^2.(16) + \ (0.50)^2.(4) + \ 2(0.50)(0.50)(0.90) = 5.45$

Portfolio (Girdhari and Murari) Variance :

$= (0.50)^2.(16) + \ (0.50)^2.(16) + \ 2(0.50)(0.50)(0.70) = 8.35$

Portfolio (Murari and Banwari) Variance :

$= (0.50)^2.(16) + \ (0.50)^2.(4) + \ 2(0.50)(0.50)(0.20) = 5.10$

Q. No. 25 : Equity shares of G Ltd, B Ltd and M Ltd have same expected return. Using the following variance-covariance table, suggest whether to invest in only G , only B, only M, equal amount in G&B, equal amount in G&M or equal amount in B&M.

	Equity shares of G	Equity shares of B	Equity shares of M.
G Ltd	1.50	0.80	0.90
B Ltd	0.80	1.20	−0.10
M Ltd.	0.90	−0.10	1.30

Answer

Portfolio Variance of G and B :

$= (0.50)^2.(1.50) + (0.50)^2.(1.20) + 2(0.50)(0.50)(0.80) = 1.075$

Portfolio Variance of G and M:

$= (0.50)^2.(1.50) + (0.50)^2.(1.30) + 2(0.50)(0.50)(0.90) = 1.15$

Portfolio Variance of Murari and Banwari :

$= (0.50)^2.(1.30) + (0.50)^2.(1.20) + 2(0.50)(0.50)(-0.10) = 0.575$

Variance of G = 1.50

Variance of B = 1.20

Variance of M = 1.30

Invest equal amount in Murari and Banwari.

Portfolio Beta

= Weighted average of Betas of the Securities constituting the portfolio. Weights being Proportions of Investment.

Example: Suppose an investor invests 40% of his funds in security A and 60% of his funds in security B. Beta of A = 1.20. Beta of B = 1.50.

Beta of Portfolio = 0.40(1.20) + 0.60(1.50) = 13.80

Minimum Risk Portfolio:

For Minimum risk portfolio (also called as minimum variance portfolio, also called as minimum SD portfolio)

$$W1 = \frac{(SD_2)^2 - r(SD_1)(SD_2)}{(SD_1)^2 + (SD_2)^2 - 2r(SD_1)(SD_2)}$$

(Mathematical derivation of this formula is given in Appendix B)

Q. No. 26: (a) You are supplied the following information regarding equity shares of the two companies :

	Kanhai Ltd.	Radhika Ltd.
Average Return	12 %	15 %
SD of return	6 %	3 %

Coefficient of correlation between returns from equity shares of Kanhai Ltd and Radhika Ltd. = 0.50

An investor is interested in investing Rs.15,00,000 in these two securities. Suggest the portfolio to minimize the risk.

Answer (a)

If r = 0.50 : Let Kanhai Ltd. = 1 Let Radhika Ltd. = 2

$$W1 = \frac{(SD_2)^2 - r(SD_1)(SD_2)}{(SD_1)^2 + (SD_2)^2 - 2r(SD_1)(SD_2)}$$

$$= \frac{(.03)^2 - (0.50)(.06)(.03)}{(.06)^2 + (.03)^2 - 2(0.50)(.06)(.03)} = 0$$

Invest total amount of Rs.15,00,000 in the equity shares of Radhika Ltd.

If the coefficient of correlation is -1, we may not apply this formula. In this case, the same result, that we get from this formula, can be obtained through the reverse ratio of the SDs. For example if the SD of A is 1 and that of B is 3, r = -1, for minimum risk variance the investment may be made in the ratio of 3:1 i.e. 75% of the funds may be invested in A and 25% in B. (We shall get the same result if we apply the above formula, but that will be time consuming)

Remember that the concept of the reverse ratio of the SDs is applicable only when r = -1.

Q. No. 26 : **(b)** How your answer will change if r = -1?

Answer

If r = -1, for minimum risk variance the investment may be made in the ratio of 3:6 i.e. 1/3rd of the funds may be invested in A and 2/3rd in B i.e. Rs.5,00,000 may be invested in A and Rs.10,00,000 in B.

Q. No. 27 : The securities A and B have the expected returns and standard deviations given below. Correlation between expected returns in 0.10.

	Return	S.D.
A	10%	20
B	20%	10

(i) Compute the return and risk, for a portfolio of A 70 per cent & B 30 per cent. Find the gain of the portfolio. Suggest the minimum risk portfolio.(ii) Revise your answers assuming that r is -1.00 instead of 0.10.

Answer

(i) Return of the portfolio = (0.70x10) + (0.30x20) = 13%

Portfolio Risk(SD)=

$$\sqrt{(0.70)^2.(20)^2 + (0.30)^2.(10)^2 + 2(0.70)(0.30)(0.10)(20)(10)} = 14.61$$

Gain of portfolio (%) =

$$= \frac{\text{Weighted Av. SD} - \text{Portfolio SD}}{\text{Weighted average SD}} \times 100$$

17 – 14.61

$$\text{Gain of portfolio (\%)} = \frac{}{17} \times 100 = 14.06$$

Minimum Risk portfolio :

$$W1 = \frac{(SD_2)^2 - r(SD_1)(SD_2)}{(SD_1)^2 + (SD_2)^2 - 2r(SD_1)(SD_2)} = \frac{(10)^2 - (0.10)(20)(10)}{(20)^2 + (10)^2 - 2(0.10)(20)(10)} = 0.1739$$

For minimum risk, the investment in A and B should be in the proportion of 0.1739 and 0.8261 respectively.

(ii) Return of the portfolio = (0.70x10) + (0.30x20) = 13%

Portfolio Risk (SD) =

$$\sqrt{(0.70)^2.(20)^2 + (0.30)^2.(10)^2 + 2(0.70)(0.30)(-1)(20)(10)} = 11$$

Gain of portfolio (%) =

$$= \frac{\text{Weighted Av. SD} - \text{Portfolio SD}}{\text{Weighted average SD}} \times 100$$

$$\text{Gain of portfolio (\%)} = \frac{17 - 11}{17} \times 100 = 35.29$$

Minimum Risk portfolio: r = −1. In this case, for minimum risk portfolio, the investment should be made in the reverse ratio of the SDS. The investment should be in the ratio of 1:2 in A and B respectively i.e. 1/3 of the total investment should be in A and 2/3 of the total investment should be in B.

Q. No. 28 : Find the maximum and minimum portfolio Standard Deviation for varying levels of coefficient of correlation between the following two securities assuming that the investments are in the ratio of 6:9 :

	Return	S.D.
A	10%	20
B	20%	10

Answer

Portfolio SD is maximum when r = +1 . (In this case the portfolio SD is equal to weighted average SD)

Maximum Portfolio SD = 20 x (0.40) + 10 x (0.60) = 14

Portfolio SD is minimum when r = −1

Minimum portfolio SD

$$= \sqrt{(0.40)^2(20)^2 + (0.60)^2(10)^2 + 2(0.40)(0.60)(-1)(20)(10)} = 2$$

Q. No. 29. The coefficient of correlation between returns of two securities, A & B, is 0.06. There standard deviations are 0.06 and 0.09 respectively. Calculate the % of diversification gain if a portfolio is constituted of these two securities with weights of 0.40 and 0.60 respectively.

Answer

Weighted average SD = 0.06x0.40 + 0.09x.60 = 0.078

Portfolio SD= 0.060

Gain of portfolio (%) = [(0.078 -0.060)/0.078]x100 = 23.08

Q. No. 30. A Ltd. has an expected return of 22% and standard of 40%. B Ltd has an expected return of 24 % and standard of 38 %. A Ltd. has a Beta of 0.86 and B Ltd a Beta of 1.24. The correlation coefficient between the return of A Ltd. and B Ltd is 0.72. The standard deviation of the market return is 20 %. Suggest: (i) Is investing in B Ltd. better than investing in A Ltd? (ii) If you invest 30% in B Ltd. and 70% in A Ltd., what is your expected rate of return and portfolio standard deviation? (iii) What is the market portfolio's expected rate of return and how much is the risk free rate? (iv) What is the beta of portfolio A's weight is 70% and B' weight is 30%?**(May, 2002)**

Answer : (i) Yes, investing in B is better than investing in A as B has higher return and lower risk. (Security B dominates security A, every rational investor will prefer B as compared to A)

(ii) (a) Expected rate of return =
(0.70x22) + (0.30x24) =22.60%

(ii) (b) Portfolio SD =
$(.70)^2(.40)^2 + (.30)^2(.38)^2 +$
$2(.70)(.30)(.72)(.40)(.38) = 37.062$

(iii)

22 = RF + .86(RM-RF) ……..(1)

24 = RF + 1.24(RM-RF) ……(2)

solving the equations, (RM-RF) = 5.263

putting this value in (1), RF =17.474 & RM = 22.737

(iv) Beta of portfolio=
(0.86 x 0.70 + 1.24 x 0.30) = 0.974

Q. No. 31 : XYZ Limited pays no taxes and is entirely financed by equity shares. The equity share has a beta of 0.6, and is priced to offer an expected return of 20 per cent, XYZ Ltd. now decides to buy back half of the equity shares by borrowing an equal amount. If the debt yields a risk free return of 10%, calculate:

(i) The beta of the equity shares after the buyback.
(ii) The required return and risk premium on the equity shares before the buyback.
(iii) The required return and risk premium on the e. shares after the buyback.
(iv) The required return on debt.
(v) The percentage increase in expected earnings per share.
(vi) The new price-earning multiple.

Assume that the operating profit of the firm is expected to remain constant in perpetuity. **(May, 2002)**

Answer :

Beta of firm after buy back = 0.60

$$0.60 = D.\beta \times \frac{D}{D+E} + E.\beta \times \frac{E}{D+E}$$

$$0.60 = 0 \times \frac{0.50}{.50+.50} + E.\beta \times \frac{0.50}{.50+.50}$$

Equity β = 1.20

(ii) Required return = 20 %
Risk free return = 10 %

Risk premium = 10 %

(iii) As β has been doubled, risk has been doubled, therefore risk premium = 20 %
So, required return = RF + risk premium
= 10% + 20% = 30%

(iv) Required return on debt = 10 %

(v) As the required rate has increased by 50%, the EPS will also increase by 50%. The new EPS is Rs.30.

(vi) Price earning ratio : Before buy back = 100/20 = 5
After buy back = 100/30 = 3.33

Q. No. 32: As an investment manager you are given the following information:

	Initial Price Rs.	Dividends Rs.	Market Price at the end of the year Rs.	Beta risk factor
Cement Ltd.(E.Shares)	25	2	50	0.8
Steel Ltd. (E. Shares)	35	2	60	0.7

Liquor Ltd. (E. Shares)	45	2	135	0.5
Govt. of India Bonds	1,000	140	1,005	0.99

RF = 14% You are required to calculate:

(i) Expected rate of return of investment in each using CAPM

(ii) Average return of portfolio. **(May, 1996)**

Answer

Assumption: There are only these four securities available in the market.

$$RM = \frac{52 + 62 + 137 + 1145}{25 + 35 + 45 + 1000} - 1 = .2633$$

= 26.33%

Security	Expected return
E.Cement Ltd	14+ 0.80(26.33 -14) =23.864
E.Steel Ltd	14+ 0.70(26.33 -14) =22.631
E.Liquor Ltd.	14+ 0.50(26.33 -14) =20.165
GOI Bonds	14+ 0.99(26.33-14) =26.2067

Average return of portfolio

(Assumption : equal amount is invested in each security)

Return (X)	W	XW
23.864	0.25	5.966
22.631	0.25	5.65775
20.165	0.25	5.04125
26.2067	0.25	6.551675
		23.216675

$$\text{Average return} = \frac{\sum XW}{\sum W} = \frac{23.216675}{1} = 23.22\ \%$$

Q. No.33: Your client is holding the following securities :

	Cost (Rs.)	Dividend/ interest	Market price	Beta
E. shares of Gold Ltd.	10,000	1725	9800	0.60
E. shares of silver Ltd.	15,000	1000	16200	0.80
E. shares of Bronze Ltd.	14,000	700	20,000	0.60
GOI Bonds	36,000	3600	34,500	1.00

Average return of the portfolio is 15.75%. Calculate expected return of each security using CAPM. (ii) RF, **(Nov. 2005)**

Answer

We have to apply CAPM, for this we require RM which is not given in the question. To calculate RM, let's assume that in the stock market only four securities are there (i) E shares of Gold (ii) E shares of Silver (iii) E shares of Bronze and (iv) GOI

BONDS. We further assume that their numbers are equal i.e. the number of E shares of Gold = the number of E shares of Silver = the number of E shares of Bronze = the number of GOI Bonds.

Wealth in the beginning of the year =
10,000+ 15,000+ 14,000+ 36,000 = 75,000

Wealth at the end of the year =
1,725 + 1,000+ 700+ 3,600+ 9,800+ 16,200+ 20,000+ 34,500 = 87525

Wealth ratio of the market = 87525/75000 = 1.167

Rate of return on market portfolio = RM = 16.70%

Assume that the portfolio referred here is being constituted by these four securities and the amount of investment in all the four securities is equal. Beta of the portfolio in this case : (0.60)(.25) + (0.80)(.25) + (.60)(.25) + (1)(.25) = 0.75

Average return of the portfolio[7] = RF + Beta(RM –RF)

15.75 = RF + .75 (16.67 – RF)

Solving the equation, RF = 12.99%

Expected return on equity shares of :		
Gold	12.99 + 0.60(16.70 –12.99) =	15.216
Silver	12.99 + 0.80(16.70 –12.99) =	15.958
Bronze	12.99 + 0.60(16.70 –12.99) =	15.216
GOI Bonds	12.99 + 1(16.70 –12.99) =	16.70

Q. No. 34 : Following is the data regarding six securities:

	A	B	C	D	E	F
Returns (%)	8	8	12	4	9	8
Risk (%) (Standard Deviation)	4	5	12	4	5	6

(i) Which of the securities will be selected?

(ii) Assuming perfect correlation, analyze whether it is preferable to invest 75% in security A and 25% in security C. **(NOV.96)**

Answer (i)
SET A:
(a) A, B and F have same return but A's SD is least. Hence, B and F are rejected.

[7] The term refers to average expected return of the portfolio

(b) Now we are left with A, C, D and E. A and D have same SD but D's return is lower. Hence D is rejected. We are left with A, C and E.

SET B:

Security	Coefficient of variation
A	(4/8) x 100 = 50%
C	(12/12) x 100 = 100%
E	(5/9) x 100 = 55.56%

The securities may be selected in the following order:
(i) A (ii) E (iii) C

Answer (ii)
Portfolio return = 9% SD = 6%
The investment may not be made in this portfolio as a better investment opportunity (investment in E) is available.

Q. No.35
The results of four portfolio managers for a 5-years period are as follows: (RF 10%, RM 16%)

Portfolio Manager	Average Return (%)	Beta
Warren	14	0.80
King	17	1.05
Tony	17	1.25
Gates	15	0.90

Select the manager with best performance.

Teaching note: Expected return v/s likely return : The term expected return, in RISK AND RETURN chapter, is used to convey two meanings:

(I) Likely return i.e. what an investor is likely to get i.e. the investor hopes to get (It is estimated on the basis of past returns) It is also referred as average return (of past years). Given probabilities, it is estimated on the basis of probabilities.

(II) Expected return i.e. i.e. what an investor deserves to get in view of risk he has taken (it is equal to RF + RISK PREMIUM) It is also required return. Risk premium may be calculated on the basis of Beta or SD.

How to identify whether an expected return given in the question is expected (Likely) or Expected (Risk based)? The answer is simple. If it has been calculated on the basis of Beta or SD it is expected (risk based) otherwise it is Expected (Likely).

Answer

	Expected return (risk based)	Expected return (average return based on historical returns)	Comment
Warren	10+ 0.80(16-10)= 14.80	14	Performance not satisfactory
King	10+ 1.05(16-10)= 16.30	17	Performance satisfactory
Tony	10+ 1.25(16-10) =17.50	17	Performance not satisfactory
Gates	10+ 0.90(16-10)= 15.40	15	-----do------

King's performance is the best.

Q. No. 36 :
The following data relate to four different portfolios:

Portfolio	Expected Rate of Return	S.D. of Returns from Portfolios
A	11%	6.00
B	14%	7.50
C	10%	3.00
D	15%	9.00

The expected return on market portfolio is 8.50 percent with a standard deviation of 3. The RF is 5 per cent. Comment on each of these portfolios.

Answer

	Expected return (risk based)	Expected return (average return based on historical returns)	Comment about performance
A	5+ (3,50)(6/3) = 12	11%	Not satisfactory
B	5+ (3,50)(7.50/3) = 13.75	14%	Satisfactory
C	5+ (3,50)(3/3) = 8.50	10%	Satisfactory
D	5+ (3,50)(9/3) = 15.50	15%	Not satisfactory

<u>REDUCING THE RISK OF A PORTFOLIO</u>:
There are two methods of reducing the risk of a portfolio:
- (i) Investing a part of investible funds in risk free securities.
- (ii) Futures contracts

- **Investment in Risk free securities**

Q. No. 37 : A senior citizen has Rs.500000 to invest. He wants to invest this amount in different securities. He wants that Beta of his portfolio should be 0.90. He selected 5 securities having average Beta of 1.20. How he can weight his portfolio to reach his target Beta?

Answer :

INVESTMENT	β	WEIGHT
FIVE SECURITIES	1.20	W_1
RISK FREE	0	W_2

$$\text{PORTFOLIO } \beta = \frac{\beta_1 \text{ X } W_1 + \beta_2 \text{ X } W_2}{W_1 + W_2}$$

$$0.90 = \frac{(1.20 \text{ X } W_1) + (0 \text{ X } W_2)}{1}$$

Solving the equation, $W_1 = .75$, $W_2 = .25$

Q. No. 38: An investor owns the following investments :

(i) 1 million equity shares of Madhav Ltd price Rs. 40 Beta 1.10

(ii) 2 million equity shares of Keshav Ltd price Rs. 30, Beta 1.20

(iii) 3 million equity shares of Bihari Ltd. pric4 Rs. 10, Beta 1.30

The investor wants to reduce the Beta of his portfolio to 0.80. Suggest.

Answer : Calculation of Beta of the investor's investments :

Name of invest.	Beta (X)	Weight (W)	XW
Madhav	1.10	40/130 = .3077	0.3385
Keshav	1.20	60/130 = .4615	0.5538
Bihari	1.30	30/130 = .2308	0.3000
		1	1.1923

To reduce the beta to 0.80:

% Required reduction in risk =[(1.1923 – 0.80) / 1.1923] x 100 = 32.90%

The investor should sell 32.90% of above portfolio and invest the sale proceeds in risk free securities (Beta of risk free securities is 0).

Sale proceeds :

Madhav	40m x .329 = 13.16m		
Keshav	60m x .329 = 19.74m		
Behari	30m x .329 = 9.87m		42.77m

Calculation of Beta in the changed scenario

Invest.	Beta (X)	Amount of investment	Weight	XW
Madhav	1.10	40m-13.16m =26.84m	26.84/130	0.2271
Keshav	1.20	60m-19.74m =40.26m	40.26/130	0.3717
Bihari	1.30	30m- 9.87m = 20.13m	20.13/130	0.2013
Risk free	0	42.77m	42.77/130	0
Total		130m	1	0.8001

Beta of the new scenario is 0.8001 which is as good as 0.8000.

❖ Futures

Q. No. 39: Madhav 's portfolio consists of the following securities :

Name of security	No.	Price	Beta
Equity shares of Hari Ltd.	1m	40	1.10
Equity shares of Nand Ltd	2m	30	1.20
Equity shares of Brij Ltd	3m	10	1.30

The index future is quoted at Rs.1350. Suggest the way of reducing the Beta of Madhav's investments to 0.80. Contract size of Index futures is 100 units. Index is the representative of the market. Madhav does not want to invest in risk free securities.

Answer : To reduce the beta to 0.80:

1. **%** Required reduction in risk =
   [(1.1923 – 0.80) / 1.1923] x 100 = 32.90%
2. To reduce the Beta to 0.80, Madhavji should make 32.90% of his portfolio i.e. the investment of Rs.130m x .3290 i.e. Rs.42.77m as risk-free.
3. For this purpose he should sell index futures of Rs.42.77m x 1.1923 i.e. Rs.50.99m.
4. The amount of one contract of index future (consisting of 100 units)is Rs.1,35,000.
5. Madhavji should sell 50.99m/135000 i.e. 377.70 say 378 contracts.

Suppose the share market declines by 10%.

- The loss on the shares will be 11.923% i.e. .11923X130m i.e. Rs.15.50m.

- The profit on futures will be 135 x 100 x 378 i.e. 51,03,000 i.e. 5.103m.

- Net loss on both the positions = 15.5m −5.103m i.e. 10.397 m. This is approximately equal to 8% of Rs.130m.

ENHANCING THE RISK OF A PORTFOLIO:

There are two methods of enhancing the risk of a portfolio:
(i) Borrowing funds and investing in risky securities
(ii) Futures contracts

A natural query: why the investor would like to enhance his risk? The desire for higher return entices him to go for enhancing the risk (in the hope of getting higher return.)

Borrowing Funds and Investing in Risky Assets

Q. No. 40 : Risk free rate of return is 8%.The return from market portfolio is expected to be 16%. The standard deviation of expected returns from market portfolio is 10%. Calculate the expected return and standard deviation of following portfolios:(i) All funds are invested in market portfolio,(ii) All funds are invested in risk free securities,(iii) 40% of funds are invested in risk free securities and 60 % in market portfolio and (iv) the investor borrows an amount equal to 20% of his funds and invests all funds (his own as well as borrowed finds) in market portfolio.

Answer :

(i) Return : 16% SD = 10%

(ii) Return : 8% SD = 0%

(iii) Return : (0.40x8) + (0.60x16) = 12.80%

$\text{Variance}=(0.40)^2.(0)^2 + (0.60)^2.(0.10)^2 + 2(0.40)(0.60)(r_{12})(0)(10) = 0.0036$
SD = 6%

(iv) $w_1 = 1.20$ $w_2 = -0.20$

Return = (1.20)(16) + (−0.20)(8) = 17.60

$\text{Variance}=(1.20)^2.(0.10)^2 + (-0.20)^2.(0)^2 + 2(1.20)(-0.20)(r_{12})(10)(0) = 0.0144$
SD = 12%

Q. No. 41 : An investor has Rs.1,00,000 to invest. He wants to obtain a return of 20 % on this amount. State the course of his action if Return on the market portfolio is 18 % and Risk free rate of return is 13 %. If the SD of the market portfolio is 5, what is the standard deviation of his action?

Answer

- Assumption: The investor can borrow at risk free rate.

- Let's borrow Rs.X and invest in the market together with Rs.1,00,000.
- Net return on Rs.1,00,000 = [(100000 + X)(.18)] –[(0.13) X] = 20,000. Solving the equation, we get X = Rs.40,000.

To calculate the overall return and risk, the borrowing is taken as negative investment, its risk is considered as zero (there is no risk in borrowing, there is risk in investing the amount of borrowing in the shares of the three companies) and its SD is taken as zero.

- W_1 = 1,40,000 / 1,00,000 = 1.40
- W_2 = –40,000/ 100000 = –0.40
- Overall return = 0.18 (1.40) + 0.13 (– 0.40) = 0.20
- Overall SD$\sqrt{[(1.40)^2.(5)^2 + (-40)^2.(0)^2 + 2(1.40)(0-.40)(r)(5)(0)]}$=7

Q. No. 42: An investor owns the following investments :

(i) 1 million equity shares of Madhav Ltd price Rs. 40 Beta 1.10

(ii) 2 million equity shares of Keshav Ltd price Rs. 30, Beta 1.20

(iii) 3 million equity shares of Bihari Ltd. pric4 Rs. 10, Beta 1.30

The investor wants to enhance the Beta of his portfolio to 1.50. Suggest.

Answer:
To increase the Beta to 1.50, the investor should borrow some money (assuming that the investor can borrow money at the rate of risk free rate of interest) and invest the same in the equity shares of the three companies (the new investment should be in the ratio of amounts of present investments).

To calculate the overall beta, the borrowing is taken as negative investment, its risk is considered as zero (there is no risk in borrowing, there is risk in investing the amount of borrowing in the shares of the three companies) and its beta is taken as zero.

% Required increase in risk = [(1.50 – 1.1923) / 1.1923] x 100 = 25.81 %

Borrowings = 130m x .2581 = 33.55 m. This amount should be invested in the shares of the three companies (the new investment should be in the ratio of amounts of present investments)

<u>Calculation of Beta in the changed scenario</u>

Investment	Beta (X)	Amount of investment	Weight (W)	XW
Madhav	1.10	40m + 10.32m = 50.32m	50.32/130	.4258
Keshav	1.20	60m + 15.48m =75.48m	75.48/130	.6967

Bihari	1.30	30m + 7.75m = 37.75	37.75/130	.3775
Borrowings	0	- 33.55m	-33.55/130	0
		130m	1	1.50

Futures

Q. No. 43 : Madhav 's portfolio consists of the following securities :

Name of security	No.	Price	Beta
Equity shares of Hari Ltd.	1m	40	1.10
Equity shares of Nand Ltd	2m	30	1.20
Equity shares of Brij Ltd	3m	10	1.30

The index future is quoted at Rs. 1350. Suggest the way of increasing the Beta to 1.50. Contract size of Index futures is 100 units. Index is the representative of the market. Your answer should be based on Index futures.

Answer : To increase the beta to 1.50:

% Required increase in risk = [(1.50 -1.1923) / 1.1923] x 100 = 25.81%

To increase the Beta to 1.50, Madhavji should purchase index futures of Rs.130m x 0.2581 x 1.1923 i.e. 40m. The amount of one contract of index future (consisting of 100 units) is Rs.1,35,000. Madhavji should purchase 40m/135000 i.e. 296 contracts.

Suppose the share market declines by 10%.
The loss on the shares will be 11.923% i.e. .11923X130m i.e. Rs.15.50m.
The loss on futures will be 13500 x 296 i.e. 3.996m
Total loss=19.496m.(This amount is approximately equal to 15% of 130m)

<u>HEDGING THE PORTFOLIO</u>

Q. NO. 44: Which positions on index future gives a speculator, a complete hedge against the following transactions:

(i) The share of Right Ltd is going to rise. He has a long position on cash market of Rs.50 Lakh on the Right Ltd. the beta of the Right Ltd. is 1.25.

(ii) The share of Wrong Ltd is going to depreciate. He has a short position on the cash market of Rs. 20 Lakhs of Wrong Ltd. the beta of Wrong Limited is 0.90.

(iii) The share of Fair Limited is going to stagnant. He has a short position on the cash market of Rs.20 Lakhs of the Fair Ltd. the beta of the Fair Ltd. is 0.75.

Answer

(i) Sell index futures Rs.62,50,000.

(ii) Purchase index futures Rs.18,00,000

(iii) Purchase index futures Rs. 15,00,000

Q. No. 45 : Makhanchor purchased 50,000 equity shares of Pootana Ltd. @ Rs. 200 on 28th Sept. 2005. Beta of his investment is 1.20. he just cannot sell these shares before 29th December, 2005. Immediately after purchasing these shares, he got adverse report about the company. He apprehends that the prices of his investment may fall. He gathers from the market that 29th December, 2005 maturity Index futures is trading in the market at 1200. Market lot of Index futures is 100 units. Suggest the action for complete hedge. Suppose he acts on your advice, what will be his profit of loss if (i) on 29th December, index falls to 1140 (ii) share price to Pootana Ltd goes down to 185 & (iii) he sells all his shares of Pootana on 29th December, 2005.

Answer

Purchase position of shares = Rs.1,00,00,000 Beta 1.20.

To protect against loss, Makhanchor should sell index futures of Rs.1,20,00,000 i.e. he should enter 100 index futures sale contracts.

Maturity:

Gain on index futures=

Rs.60 per unit x 100 units per contract x100 contracts = Rs. 6,00,000

Loss on 50000 shares = 50000 x 15 = Rs. 7,50,000

Net loss Rs. 1,50,000

Q. No. 46 : Sh Bihari ji has the following position in cash segment of the stock exchange. He is interested in reducing risk to the extent given in the following lines using Index futures. Suggest.

1. He has a long position on cash market of Rs.50 Lakh on the Right Ltd. the beta of the Right Ltd. is 1.25. He wants to reduce his risk by 20%.

2. He has a short position on the cash market of Rs. 20 Lakhs of Wrong Ltd. the beta of Wrong Limited is 0.90. He wants to reduce his risk by 10%

3. He has a short position on the cash market of Rs.20 Lakhs of the Fair Ltd. the beta of the Fair Ltd. is 0.75. He wants to reduce his risk by 20%

Answer

Sale Index in the futures market	50L x 1.25 x 0.20
Purchase Index in the futures market	20L x 0.90 x 0.10
Purchase Index in the futures market	20L x 0.75 x 0.20
Net position in Index futures	Sale 7.70L

Suppose the market falls by 10% :

Profit on futures		+ 0.77L
Loss on Rights Ltd	50L x 1.25 x 0.10 x 0.20	−1.25L
Profit on Wrong Ltd	20L x 0.90 x 0.10 x 0.10	+ 0.18L
Profit on Fair Ltd	20L x 0.75 x 0.10 x 0.20	+ 0.30L
Total profit or loss		0

Suppose the market rises by 10% :

Loss on futures		-0.77L
Profit on Rights Ltd	50L x 1.25 x 0.10 x 0.20	+ 1.25L
Loss on Wrong Ltd	20L x 0.90 x 0.10 x 0.10	- 0.18L
Loss on Fair Ltd	20L x 0.75 x 0.10 x 0.20	-0.30L
Total profit or loss		0

Q.No.47: Ram Buys 10000 shares of X Ltd at Rs.22 and obtains a complete hedge of shorting 400 Nifties at Rs.1100 each. He closes out his position at the closing price of the next day at which point the share of X Ltd. has dropped 2% and the Nifty future has dropped 1.50%. What is the overall profit/loss of the set of transaction.? **(MAY, 2005)**

Answer

Sales of Nifties = 400x1100 = Rs.4,40,000
Purchase of shares of Rs.2,20,000
Loss on shares = Rs.4,400
Profit on Nifties = Rs.6,600
Profit of the set of the transaction = Rs.2,200.

Q. No.48: NARAINI MUTUAL fund is holding 1m equity shares of Shri Ltd. The current market price is Rs.100 per share. The mutual fund manager believes that after six months the share is like to appreciate substantially but has the apprehension that for next six months it may face a declining tendency. If the apprehension materializes, the NAV of the MF scheme will come down and that may bring down the reputation of the MF. He wants to completely hedge this position through the index futures. The stock exchange is currently traded at 5000. Market lot size is 40 units. Beta value of equity shares of Shri Ltd is 0.80. Suggest the course of his action. Would this position would be risk less? What would be the beta of this hedged position?

Answer

Position in cash market: Investment (purchase) of Rs.100m equity shares.

Suggested position in derivative market for hedging: Sell futures Rs.80m.

The position would be risk less if the return on equity shares of Shri Ltd changes by 80% of the change in return of the Index and that to in the same direction.

The beta of the hedged position would be zero provided the return on equity shares of Shri Ltd changes by 80% of the change in return of the Index and that to in the same direction.

MPT seeks to construct an optimal portfolio by considering the relationship between risk and return. The theory recommends that the risk of a particular security should not looked on a standalone basis, rather it should be looked in relation to portfolio. A security may be very risky but when combined with some other security, the combination may not be that much risky (because some of its negative fluctuations may be set off by positive fluctuations of the other security)

The theory explores how an investor can use diversification to optimize his portfolio.[8] Markowitz showed that, using portfolio, the risk can be minimized without reducing the expected return, by diversifying out the unsystematic risk. Consider two securities, A and B. A's expected return is 10% and SD is 15%, B's expected return is 20%, SD is 30%. Coefficient of correlation between the returns of the two assets is 0. An investor is planning to invest all his money in A because he finds that he cannot bear the risk of B (because B's risk is quite high). A portfolio manager suggests him that he may invest 80% of his funds in A and 20% of funds in B. This will enhance his return to 12% and risk will be reduced to 13.40%.

The theory suggests three steps for portfolio decisions:

(I) Select the securities of your choice[9] and Construct all possible portfolios.

(II) Classify these portfolios into two parts (a) Efficient[10] and (b) Inefficient[11]. A portfolio is classified as efficient one if it is not inefficient.

A portfolio is inefficient if :

(a) it is rejected by application of Set A, or

(b) there exists some other portfolio which is better than this portfolio in both the respects. i.e. it has higher return and lower risk.

When the efficient portfolios are plotted in risk-return space, we get efficient frontier. A set of all efficient portfolios is known as efficient frontier[12] . As per MPT, an investor should seek a portfolio that lies on the efficient set.

---

[8] By optimizing the portfolio we mean : constructing a portfolio that gives maximum return for an acceptance level of risk .

[9] Here the investor can take the help of Set A given in the beginning of the chapter.

[10] A portfolio is efficient if

(a) it is not rejected by application of Set A, and

(b) there exists no other portfolio which has higher return and lower risk.

[11] " A portfolio is not efficient if there is another portfolio with a higher expected return and a lower standard deviation, a higher expected return and same standard deviation, or same expected return but a lower standard deviation."- Van Horne

[12] The efficient frontier is the set of portfolios on the upper left boundary of the attainable set, between the minimum variance portfolio and the maximum return portfolio.

(III) Select the portfolio on efficient frontier. For this purpose, a tangent should be drawn of the efficient frontier starting from the point of Risk Free rate of return on Y-Axis.

Q. No. 49 : Assuming r between returns from two securities A and B to be -1, Find portfolio return and portfolio SD using the following data : Security A : Mean return 12, SD 16; Security B: mean return 16 SD 20.You may assume that portfolio consists of (i) 100% in A and 0 in B (ii) 90% in A and 10% in B (iii) 80% in A and 20% in B………(xi) 0% and 100% in B. Which of these portfolios are inefficient? How your answer will change if (a) r = +1, (b) r=0, (c) r= +0.5 and (d) =-0.50. Draw the portfolios on Risk and return Chart.

Answer

Port-folio No.	Invest-ment in A %	Invest-ment in B %	Port-folio Return%	Port-folio SD if r= -1	Whether inefficient
1	100	0	12.00	16.00	Yes, Some other portfolio (No.2) offers higher return with lower risk
2	90	10	12.40	12.40	Yes, Some other portfolio (No.3) offers higher return with lower risk
3	80	20	12.80	8.80	Yes, Some other portfolio (No.4) offers higher return with lower risk
4	70	30	13.20	5.20	Yes, Some other portfolio (No.5) offers higher return with lower risk
5	60	40	13.60	1.60	No as it is neither rejected by application of Set A nor there exists some other portfolio witch has higher return and lower SD.
6	50	50	14.00	2.00	----------do-------
7	40	60	14.40	5.60	----------do-------
8	30	70	14.80	9.20	----------do-------
9	20	80	15.20	12.80	----------do-------
10	10	90	15.60	16.40	----------do-------
11	0	100	16.00	20.00	----------do-------

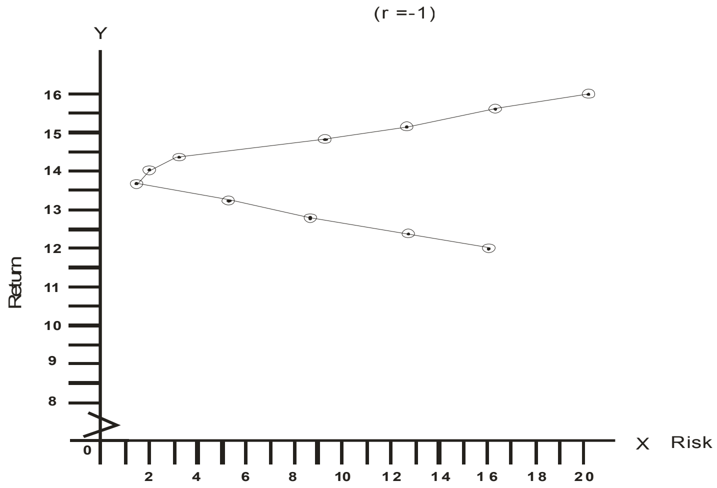

Diagram 2

Answer (a)

Port-folio No.	Invest-ment in A %	Invest-ment in B %	Port-folio Return %	Port-folio SD if r = + 1	Whether inefficient
1	100	0	12.00	16.00	No as it is neither rejected by application of Set A nor there exists some other portfolio witch has higher return and lower SD.
2	90	10	12.40	16.40	----------do------------
3	80	20	12.80	16.80	----------do------------
4	70	30	13.20	17.20	----------do------------
5	60	40	13.60	17.60	----------do------------
6	50	50	14.00	18.00	----------do------------
7	40	60	14.40	18.40	----------do------------
8	30	70	14.80	18.80	----------do------------
9	20	80	15.20	19.20	----------do------------
10	10	90	15.60	19.60	----------do------------
11	0	100	16.00	20.00	----------do------------

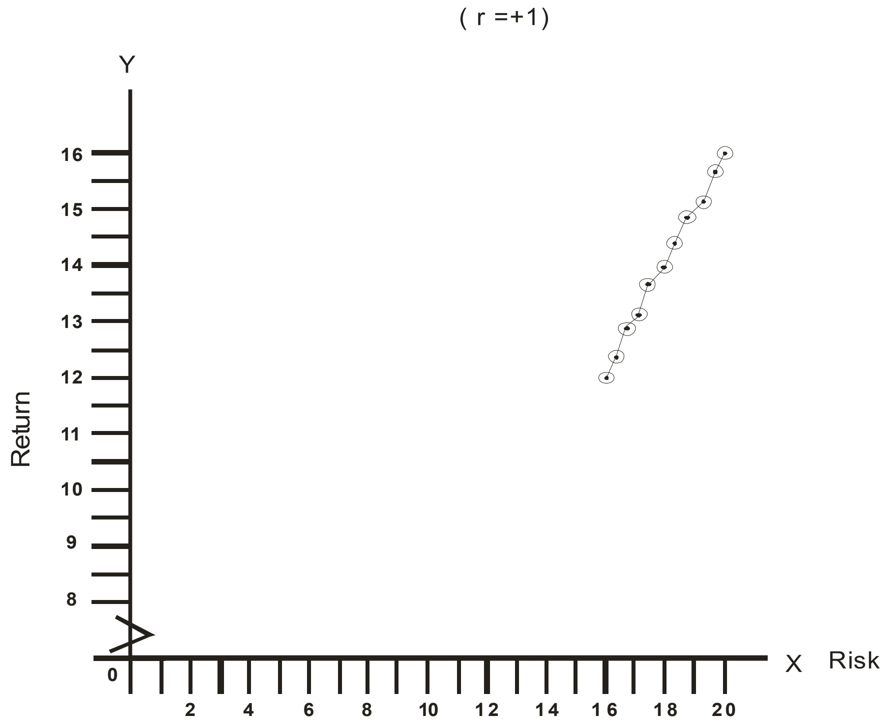

Diagram 3

Answer (b)

Portfolio No.	Investment in A %	Investment in B %	Portfolio Return %	Portfolio SD if **r = 0**	**Whether inefficient**
1	100	0	12.00	16.00	Yes, Some other portfolio (portfolio No.2) offers higher return with lower risk
2	90	10	12.40	14.54	Yes, Some other portfolio (portfolio No.3) offers higher return with lower risk
3	80	20	12.80	13.41	Yes, Some other portfolio (portfolio No.4) offers higher return with lower risk
4	70	30	13.20	12.71	Yes, Some other portfolio (portfolio No.5) offers higher return with lower risk
5	60	40	13.60	12.50	No as it is neither rejected by application of Set A nor there

					exists some other portfolio witch has higher return and lower SD.
6	50	50	14.00	12.81	----------do----------
7	40	60	14.40	13.60	----------do----------
8	30	70	14.80	14.80	----------do----------
9	20	80	15.20	16.32	----------do----------
10	10	90	15.60	18.07	----------do----------
11	0	100	16.00	20.00	----------do----------

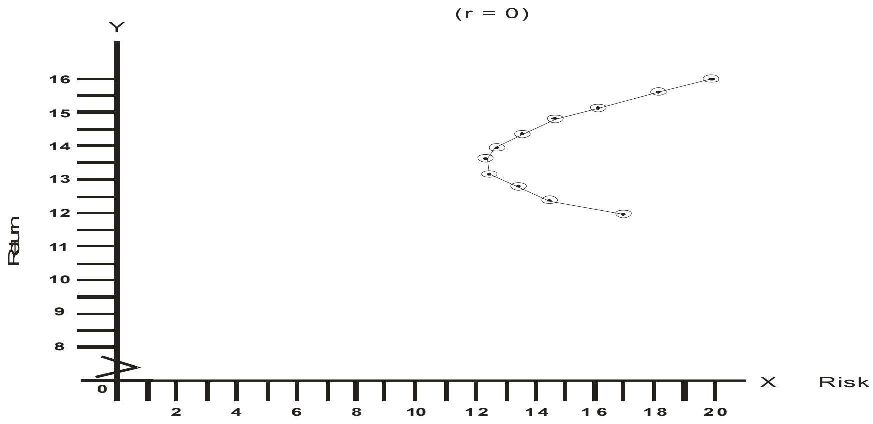

Diagram 4

Answer (c)

Port-folio No.	Invest-ment in A %	Invest-ment in B %	Port-folio Return %	Port-folio SD if r= +0.5	Whether inefficient
1	100	0	12.00	16.00	Yes, Some other portfolio (portfolio No.2) offers higher return with lower risk
2	90	10	12.40	15.50	Yes, Some other portfolio (portfolio No.3) offers higher return with lower risk
3	80	20	12.80	15.20	Yes, Some other portfolio (portfolio No.4) offers higher return with lower risk
4	70	30	13.20	15.12	No as it is neither rejected by application of Set A nor there exists some other portfolio witch has higher return and lower SD.

5	60	40	13.60	15.26	----------do---------
6	50	50	14.00	15.62	----------do---------
7	40	60	14.40	16.18	----------do---------
8	30	70	14.80	16.92	----------do---------
9	20	80	15.20	17.82	----------do---------
10	10	90	15.60	18.85	----------do---------
11	0	100	16.00	20.00	----------do---------

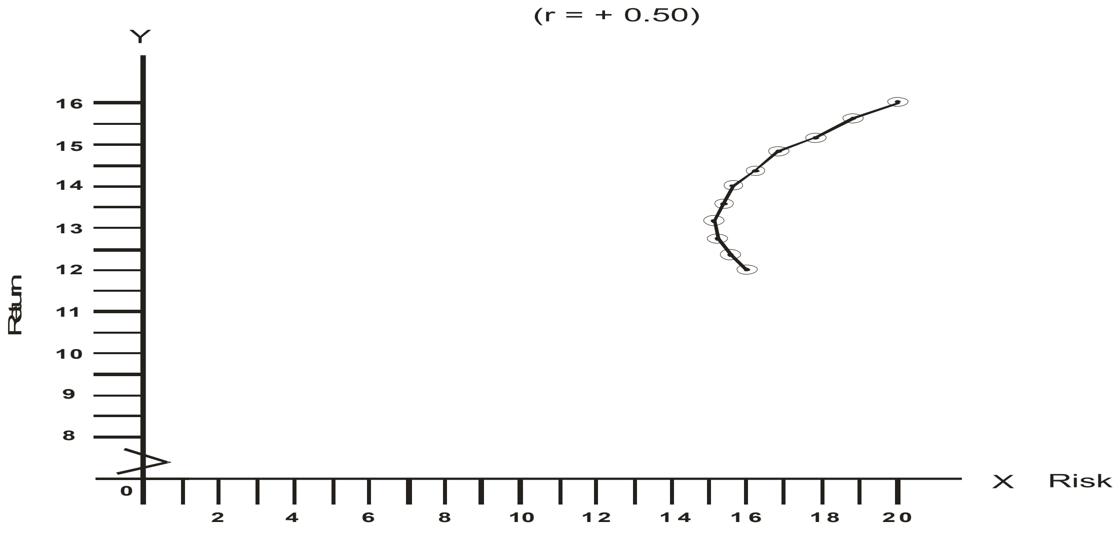

Diagram 5

Answer (d)

Port-folio No.	Invest-ment in A %	Invest-ment in B %	Port-folio Return %	Port-folio SD if **r= − .5**	**Whether inefficient**
1	100	0	12.00	16.00	Yes, Some other portfolio (portfolio No.2) offers higher return with lower risk
2	90	10	12.40	13.51	Yes, Some other portfolio (portfolio No.3) offers higher return with lower risk
3	80	20	12.80	11.34	Yes, Some other portfolio (portfolio No.4) offers higher return with lower risk
4	70	30	13.20	9.71	Yes, Some other portfolio (portfolio No.5) offers higher return with lower risk

5	60	40	13.60	8.91	No as it is neither rejected by application of Set A nor there exists some other portfolio witch has higher return and lower SD.
6	50	50	14.00	9.17	----------do-----------
7	40	60	14.40	10.40	----------do-----------
8	30	70	14.80	12.32	----------do-----------
9	20	80	15.20	14.66	----------do-----------
10	10	90	15.60	17.26	----------do-----------
11	0	100	16.00	20.00	----------do-----------

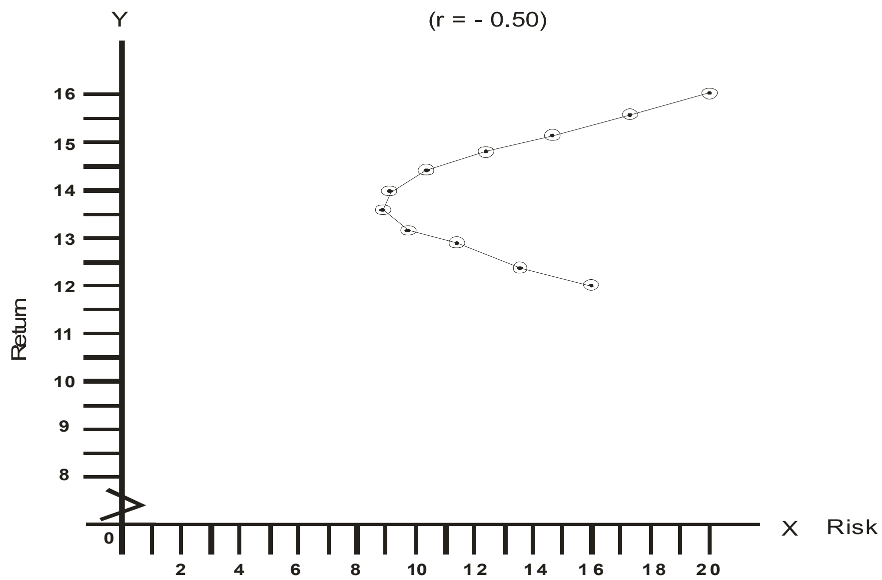

Diagram 6

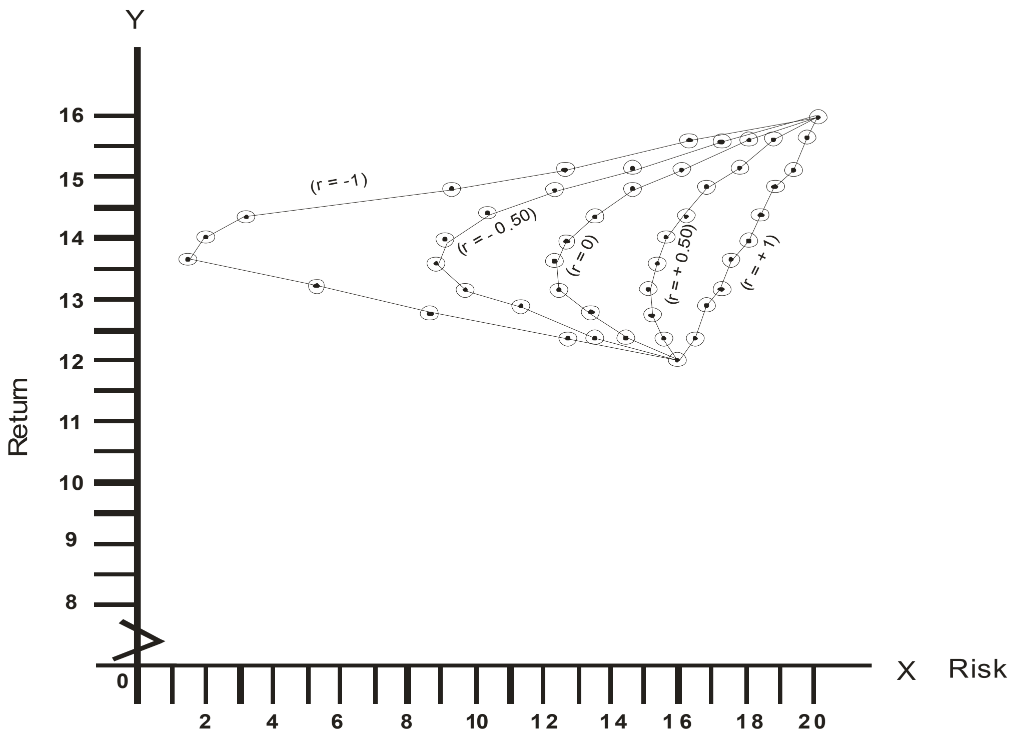

Diagram 7

Q. No.50: Plot the following portfolios on a graph;

	A	B	C	D	E	F	G	H
Expected return	10	13	14	16	18	19	19	20
SD	23	20	24	30	30	32	36	43

Which of the portfolios are efficient?

(a) Suppose you can borrow and lend at 10%. Which of the portfolio is the best?

(b) You can't borrow or lend. You want that the SD of your investment should not be more than 29. What is the maximum return you could expect?

(c) You can borrow or lend at 10%. You want that the SD of your investment should not be more than 28. What is the maximum return you could expect?

Answer

Portfolio No.	Portfolio Return %	Portfolio SD	Whether inefficient
A	10	23	Yes, because B's return is higher and risk is lower as compared to this security
B	13	20	No, as it is neither rejected by application of Set A nor there exists some other portfolio

			which has higher return and lower SD.
C	14	24	---------do-----------
D	16	30	Yes, its return is less than that of E though its risk is the same as that of E
E	18	30	No, as it is neither rejected by application of Set A nor there exists some other portfolio which has higher return and lower SD.
F	19	32	--------------do-----------
G	19	36	Yes, its return is same as that of F but risk is more than that of F.
H	20	43	No as it is neither rejected by application of Set A nor there exists some other portfolio which has higher return and lower SD.

Efficient portfolios are: B, C, E, F, H

Teaching note: Not to be given in the exam. The rate at which a party can lend as well borrow is considered to be risk free rate of interest.

(a) F **(b)** C

(c) The portfolio of our choice is F but its SD is 32 while we have to limit the SD to 28. To achieve the reduction in SD, we may invest a part of investible funds in F and other part in Risk free securities. Let's invest w_1 in F and w_2 in Risk free.

$$28 = \sqrt{(w_1)^2.(32)^2 + 0 + 0} \qquad w_1 = 0.875$$

Portfolio return = 19x0.875 + 10x0.125 = 17.875%

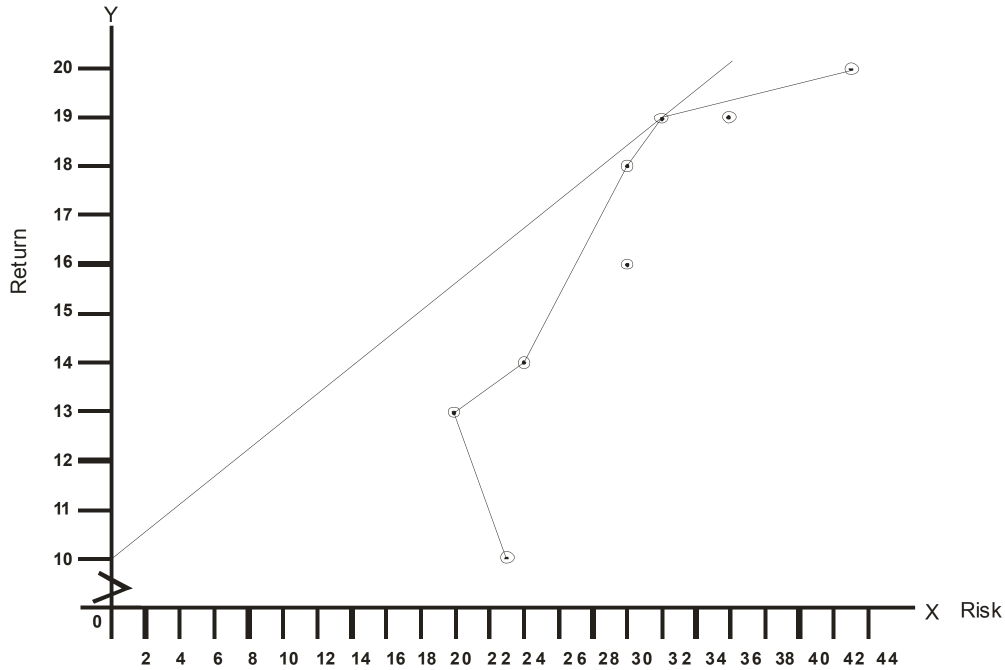

Diagram 8

Q. No. 51 : Assuming r between returns from two securities A and B to be -1, Find portfolio risk and portfolio SD using the following data : Security A : Mean return 12, SD 16; Security B: mean return 16 SD 20.You may assume that portfolio consists of (i) 100% in A and 0 in B (ii) 80% in A and 20% in B (iii) 60% in A and 40% in B (iv) 40% in A and 60% in B (v) 20% in and 80% in B and (vi) 0 in A and 100% in B.. Assuming the risk free rate of return is 8%, what amount should be invested by an investor in A. Assume that the investor's investible funds amount to Rs.10m.

Remember: There are two basic differences between Traditional Theory of portfolio and Modern Portfolio Theory (MPT).

(i) Traditional theory does not consider the coefficient of correlation between the returns of various securities; The MPT reveals that the degree of risk reduction depends upon degree of coefficient correlation between returns from different investments.

(ii) As per Traditional theory, investment decisions should be based on Mean Return and Risk (SD) of various securities; (We base our investment decisions on the basis of Coefficient of variation. Coefficient of variation is calculated using Mean and SD.

As per MPT, the investment should be taken on the basis of SD and Risk premium

Answer

Port-folio No.	Invest-ment in A %	Invest-ment in B %	Port-folio Return%	Port-folio SD if r= -1	Whether inefficient
1	100	0	12.00	16.00	Yes, Some other portfolio (portfolio No. 2) offers higher return with lower risk
2	80	20	12.80	8.80	Yes, Some other portfolio (portfolio No. 3) offers higher return with lower risk
3	60	40	13.60	1.60	No as it is neither rejected by application of Set A nor there exists some other portfolio witch has higher return and lower SD.
4	40	60	14.40	5.60	----------do------
5	20	80	15.20	12.80	----------do------
6	0	100	16.00	20.00	----------do------

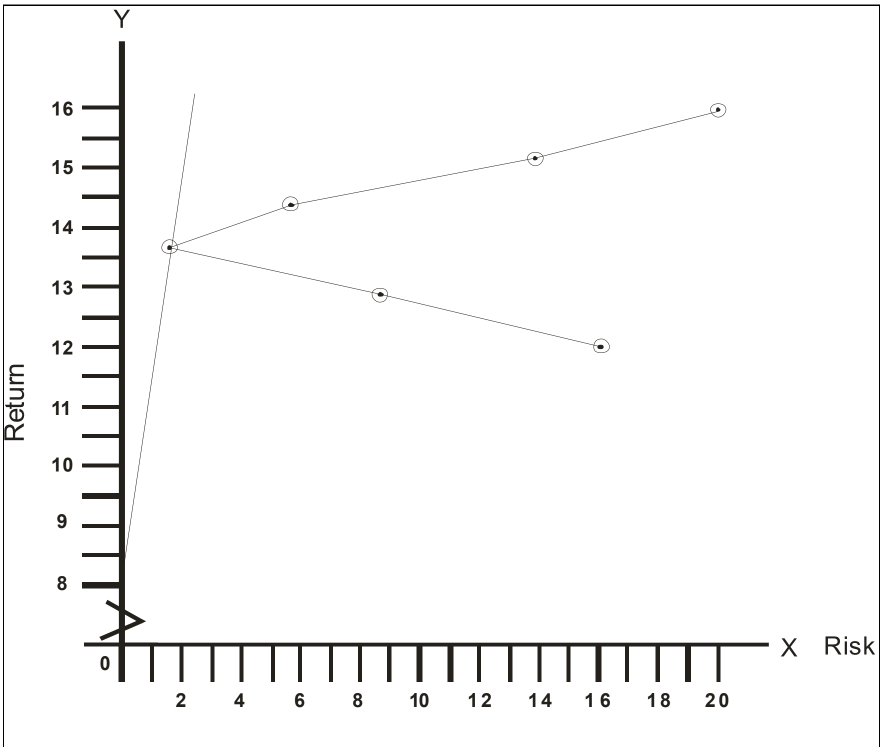

Diagram 9

SOME MORE ASPECTS OF "RISK AND RETURN

1. EFFICIENT MARKET HYPOTHESIS
2. ALPHA
3. CHARACTERISTIC LINE
4. SECURITY MARKET LINE (SML)
5. CAPITAL MARKET LINE (CML)
6. MARKET MODEL
7. MATRIX APPROACH IN INVESTMENT DECISIONS

EFFICIENT MARKET HYPOTHESIS

EMH is one of the central ideas of modern finance. Market efficiency means that the market price of a security reflects all available information. In other words, as per EMH, an efficient market responds quickly to new information, i.e., market price of security changes rapidly, completely and accurately in accordance with new information. (According to Adam Smith, "invisible hand" of the market place works quickly). "The market efficiency will produce prices that are appropriate in terms of current knowledge, and investors will be less likely to make unwise investments." The key to market efficiency is the high level of competition among participants in the market.

This implies that the new information cannot be used to create a trading strategy to beat the market i.e. the investor cannot make abnormal profits[13] in the share market (except by chance). According to the EMH, stocks always trade at their fair market value on stock exchanges, making it impossible for investors to purchase undervalued stocks or sell stocks for inflated prices. Suppose that Ranbaxy announces a new drug that could completely prevent Dengu. Suppose before the announcement the price was Rs.530, after this announcement there will be no seller at this price. The market price of shares of Ranbaxy will change immediately to a new equilibrium level[14], suppose Rs.600, (where it should be in view of new announcement).. Suppose an investor gets this information and places an order at this price believing that he will purchase the shares before this announcement's impact on the prices of the share and later on when the prices will move to Rs.600, he will sell and make an abnormal profit. The

---

[13] By abnormal profit we mean: excess profit relative to risk.

[14] The equilibrium price of a stock is determined by demand and supply forces, based on the available information. Quickly as the fresh information becomes available, a new equilibrium point is reached. The reason is that market ……
→is crowded by active, intelligent, greedy and rational investors who are competing against each other for profit maximization. The key to market efficiency is the high level of competition among participants in the market.

investor would be disappointed as share is likely to be available in the market Rs.600 only leaving no scope for abnormal profit.

An attempt to outperform the market will effectively be a game of chance rather than skill. According to the EMH, expert stock selection or market timing is of no value. Indeed, the only way an investor can possibly obtain higher than average returns is by purchasing riskier investments.

The EMH is highly controversial theory. Supporters argue it should be impossible to outperform the market. Critics always quote the case of Warren Buffett who consistently has been beating the market over long periods, which according to EMH is virtually impossible. Supporters put forward the plea that he has been taking abnormal risk.

Assumptions of EMH:
1. Information flows freely.
2. All investors have the same access to information.
3. No transaction costs.
4. Investors are rational and aim to maximizing the profit.
5. Every investor has access to lending and borrowing.

Based on different information sets, there are three forms of market efficiency.

Weak Form Efficiency

A market is weakly efficient if its prices fully reflected all historical information, i.e. market responds only to information derived from past. The current share price reflects past price, volume and rate of return information. In this type of market, the prices follow random pattern i.e. today's price is independent of yesterday's price; tomorrow's price will be independent of today's price etc. The weak form of EMH says that one cannot predict future stock prices on the basis of past stock prices.

- Implication: technical analysis is of no use.

Semi-strong Form Efficiency

A market has semi-strong efficiency if its prices at all times reflect all public information. It means it is not possible to abnormal profit by reading newspapers, looking at the company's annual accounts and so on as the prices move on generation of public information such as announcement of dividend, quarterly results, fresh public offer, new project, merger etc.

- Implication: even fundamental analysis is of no use.

Strong Form Efficiency

Market has strong efficiency if its prices fully reflect all information, both public and private. Abnormal profit can be made only on the basis of Luck/chance; skills have no role to play.

- Implications: even insider information is of no use.

Tests for weak form of efficiency:

Run Test: It is a test of randomness of price behavior, (the term randomness means that the share prices do not follow any trend/pattern) i.e. today's price is independent of yesterday's price; tomorrow's price will be independent of today's price etc. In other words, the test measures the likelihood that a series of two variables (positive and negative changes) is a random occurrence. There are two tests:

Run Test

Under this method, the sampling technique of testing the significance is applied. We find the values of three variables (i) r i.e., no. of runs (ii) n_1 (iii) n_2 . A run is defined as the repeated occurrence of the same category of return/price. n_1 refers to total number of + changes and n_2 refers total number of negative changes.

We calculate mean and SD of runs using the following formulas:

$$X = \text{mean value of } r = = \frac{2n_1 n_2}{n_{1+} n_2} + 1$$

$$SD^2 = \frac{2\, n_1 . n_2 (2n_1 . n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}$$

$SD = \sqrt{(SD)^2}$

Now we find the standard upper limit and standard lower limit of no. of runs for randomness. If our calculated mean value of r lies within the standard upper limit and standard lower limits, the randomness is there i.e. the market is weakly efficient, otherwise it is not weakly efficient.

For calculating the standard upper and lower limit, we have to refer standard values. These values are available in different tables. If $n_1 + n_2$ is less than 30, we find these values from t-table. The table is consulted for ($n_1 + n_2 -1$). [($n_1 + n_2 -1$) is referred as degree of freedom.]

If $n_1 + n_2$ is 30 or more, we refer Z table. For Z value degree of freedom is irrelevant.

Table values are available at various levels of significance, for example 5% level of significance (also referred as 95% level of confidence), 10% levels of significance (also referred as 90% level of confidence). 5% level of significance (also referred as 95% level of confidence) means that the probability of correctness of our conclusion is 95%. 10% level of significance (also referred as 90% level of confidence) means that the probability of correctness of our conclusion is 90%. (Remember that the probability of conclusions based on sampling is never 1).

[It may be of help if we remember that Z value at 5% level of significance is 1.96]
Standard upper limit = Mean + Table value x SD

Standard lower limit = Mean - Table value x SD

Example : r = 23 n_1 = 20 n_2 = 30

Mean = 25   SD =3.36

Table value of Z at 95% level of confidence : 1.96

Standard upper limit = 25 + 1.96 x 3.36 = 31.5856

Standard lower limit =25 - 1.96 x 3.36 = 18.4144

Calculated mean value of r lies within the standard upper limits and standard lower limits, the randomness is there i.e. the market is weakly efficient.

Q.No.52 The closing value of Sensex for the month of Oct., 2007 is given below:

Date	Closing Sensex Value	Date	Closing Sensex Value
1.10.07	2800	16.10.07	3300
3.10.07	2780	17.10.07	3450
4.10.07	2795	19.10.07	3360
5.10.07	2830	22.10.07	3290
8.10.07	2760	23.10.07	3360
9.10.07	2790	24.10.07	3340
10.10.07	2880	25.10.07	3290
11.10.07	2960	29.10.07	3240
12.10.07	2990	30.10.07	3140
15.10.07	3200	31.10.07	3260

You are required to test the week form of efficient market hypothesis by applying the run test at 5% and 10% level of significance.
Following value can be used:
Value of t at 5% is 2.101 at 18 degrees of freedom
Value of t at 10% is 1.734 at 18 degrees of freedom
Value of t at 5% is 2.086 at 20 degrees of freedom. Value of t at 10% is 1.725 at 20 degrees of freedom. (8 Marks)) (Nov. 2008 SFM)

Answer

$r = 8$ $n_1 = 11$ $n_2 = 8$ Mean = 10.26 SD = 2.06

5% Level of significance: Table value of t for 18 degrees of freedom at 5% level of significance : 2.101

Standard upper limit = 10.26 + 2.101 x 2.06 = 14.5881

Standard lower limit = 10.26 – 2.101 x 2.06 = 5.9319

Calculated mean value of r lies within the standard upper limits and standard lower limits, the randomness is there i.e. the market is weakly efficient.

10% Level of significance: Table value of t for 18 degrees of freedom at 10% level of significance : 1.7434

Standard upper limit = 10.26 + 1.7434 x 2.06 = 13.8514

Standard lower limit = 10.26 – 1.7434 x 2.06 = 6.6686

Calculated mean value of r lies within the standard upper limits and standard lower limits, the randomness is there i.e. the market is weakly efficient.

Autocorrelation test:

The autocorrelation test is applied to identify the degree of autocorrelation in a time series. It measures the correlation between the differences in current and lagged observations of the time series of stock returns. If the coefficient of correlation tends to zero, the randomness is there i.e. the market is weakly efficient, otherwise it is not weakly efficient.

Q.No.53 Given the data below: apply autocorrelation test for find whether the market if weakly efficient. Use time lag of 10 days:

Trading days	Closing Sensex
1	13450
2	13440
3	13430
4	13380
5	13370
6	13340
7	13330
8	13335
9	13310
10	13270
11	13250
12	13290
13	13330
14	13290
15	13300
16	13320
17	13330
18	13320
19	13300
20	13320

Calculation of Changes in index values (with time lag of 10 days)

Trading day	Closing Sensex	Change (X)	Trading day	Closing Sensex	Change (Y)
1	13450		11	13250	-
2	13440	-10	12	13290	+40
3	13430	-10	13	13330	+40
4	13380	-50	14	13290	-40
5	13370	-10	15	13300	+10
6	13340	-30	16	13320	+20
7	13330	-10	17	13330	+10
8	13335	+5	18	13320	+10
9	13310	-25	19	13300	-20
10	13270	-40	20	13320	+20

Calculation of coefficient of correlation

X	x	x^2	Y	y	y^2	xy
-10	+10	100	+40	30	900	300
-10	+10	100	+40	30	900	300
-50	-30	90	-40	-50	2500	1,500
-10	+10	100	+10	0	0	0
-30	-10	100	+20	+10	100	-100
-10	+10	100	+10	0	0	0
+5	+25	625	+10	0	0	0
-25	-05	25	-20	-30	900	150
-40	-20	400	+20	10	100	-200
$\sum X$ = -180	$\sum x = 0$	$\sum x^2$ = 1640	$\sum Y$ = +90	$\sum y = 0$	$\sum y^2$ =5400	$\sum xy$ = 1950

Covariance = $\sum xy/n$ = 1950/9 = 216.67

SD X = $\sqrt{(\sum x^2 / n)}$ = $\sqrt{(1640/9)}$ = 13.50

SD Y = $\sqrt{(\sum y^2 / n)}$ = $\sqrt{(54000/9)}$ = 24.50

$$r = \frac{\text{Covariance}}{\text{SD X . SD Y}} = \frac{215.67}{24.50 \times 13.50} = +0.65$$

As r does not tend to zero, the market is not weak.

Test for semi-strong form of efficiency:

Event test: By the term 'event' here we mean, generation of some public information, for example, dividend announcement, merger announcement, fire, winning some law suit (it was not expected) etc. Under this test we measure the impact of the event on the share prices.

Event study enables us to assess the impact of a particular event on share prices. For this purpose, we calculate abnormal return. Abnormal return is the difference between the expected return (calculated on the basis of characteristic line) return and actual return. If the total of abnormal return of difference periods tends to zero, the market is semi-strong efficient, other was it is not.

Q.No.54 X and Y follow financial year as accounting year. On 30th Sept ,2009 both the companies announced interim dividend (It was not expected by the market). Apply the event test to find whether the market is Semi-strong efficient or not. You are given the following data :

Month ending :	Actual annualized return %		Market annualized return
	X	Y	
30th June, 2009	11.23	12.73	10.45%
31st July, 2009	11.44	11.94	10.75%
31st Aug, 2009	12.05	12.55	10.95%
30th Sept. 2009	12.30	13.50	13.00%
31st Oct, 2009	12.25	13.51	13.10%
30th Nov. 2009	12.32	13.65	13.15%
31st Dec. 2009	12.35	13.85	13.05%

Company	Alpha	Beta
X	1.27%	0.95
Y	1.45%	1.05

Answer:

Expected return of Investment (on the basis of characteristic line) :

= Alpha + Beta x Market return

X

	Actual annualized return %	Expected annualized return %	Abnormal return %
30th June, 2009	11.23	1.27 + 0.95 X 10.45 = 11.20	+ 0.03
31st July, 2009	11.44	1.27 + 0.95 X 10.75 = 11.30	+ 0.14
31st Aug, 2009	12.05	1.27 + 0.95 X 10.95 = 11.67	+ 0.38
30th Sept. 2009	12.50	1.27 + 0.95 X 13.00 = 13.67	- 1.17
31st Oct, 2009	12.55	1.27 + 0.95 X 13.10 = 12.45	+ 0.10
30th Nov. 2009	12.52	1.27 + 0.95 X 13.15 = 13.76	- 1.24
31st Dec. 2009	12.57	1.27 + 0.95 X 13.05 = 13.67	- 1.10

Y

	Actual annualized return %	Expected annualized return %	Abnormal return %
30th June, 2009	12.73	1.45 + 1.05 X 10.45 = 12.42	+ 0.31
31st July, 2009	11.94	1.45 + 1.05 X 10.75 = 12.74	- 0.80
31st Aug, 2009	12.55	1.45 + 1.05 X 10.95 = 12.95	- 0.40
30th Sept. 2009	13.50	1.45 + 1.05 X 13.00 = 15.10	- 1.60
31st Oct, 2009	13.51	1.45 + 1.05 X 13.10 = 15.21	-1.70
30th Nov. 2009	13.65	1.45 + 1.05 X 13.15 = 15.26	-1.91
31st Dec. 2009	13.85	1.45 + 1.05 X 13.05 = 15.15	-1.30

Period wise average abnormal return:

	Average abnormal return
30th June, 2009	(0.03 + 0.31) / 2 = + 0.17
31st July, 2009	(0.14 - 0.80) / 2 = - 0.33
31st Aug, 2009	(0.38 - 0.40)/ 2 = - 0.01
30th Sept. 2009	(- 1.17 -1.60)/2 = - 1.385
31st Oct, 2009	(+ 0.10 -1.70)/2 = - 0.80
30th Nov. 2009	(- 1.24 - 1.91)/2 = - 1.575
31st Dec. 2009	(- 1.10-1.30) / 2 = - 1.20
Total of average abnormal return = - 5.13	

As the total of average abnormal return does not tend to zero, the semi-strong efficiency is not proved.

Test for Strong form of efficiency:

For testing the Strong form efficiency, we have to compare the " return obtained by Mutual fund managers (who are experts in the field of investment)" with "the return obtained by Insider trading (Inside trading is based on private information)" If the insiders have been able to earn higher returns, the strong form of efficiency is not proved.

Q. No. 55: X Ltd's issued capital is 4 crores equity shares of Rs.10 each. Y Ltd's issued capital is 12 crores equity shares of Rs.10 each.

On 14th March, 2005, market price per share is Rs.20 for X & Rs.30 for Y.

On 15th March, 2005, the management of Y decides at a private meeting, to make a cash takeover bid for X at a price of Rs.30 per share. The take over will result in synergy gain with a present value of Rs.20 Crores.

On 17th March, 2005, Y Ltd. publicly announces its plan for a cash take over bid. No announcement is made about synergy gain.

On 21st March. 2005, Y Ltd. submits all the documents to the regulatory authorities and makes a public announcement of all details including synergy gain.

Required: Ignoring tax and assuming the details given are the only factors having an impact on the share price of X and Y, determine the market price of share of X as well as Y on 15th March, 17th march and 21st March if market is (1) semi-strong form efficient, and (2) strong form efficient.

Answer

Date	Semi-strong		Strong	
	X	Y	X	Y
15th March	20	30	30	28.33
17th March	30	26.67*	30	28.33
21st March	30	28.33*	30	28.33

Value per share of Y = (30 x 12Crores -10 x 4Crores)/12Crores = 26.67
Value per share of Y = 26.67 + (20Crores/12 Crores) = 28.33

ALPHA It is expected (Likely) return of an investment when the return from market portfolio is zero.

Alpha = Average return of the security - (Beta x RM)
Example :
(X = return on market portfolio; Y = return on specific security)

Year	X	Y	x	y	xy	x^2
1981	15	16	0	0	0	0
1982	14	12	-1	-4	4	1
1983	17	19	2	+ 3	6	4
1984	16	18	1	+ 2	2	1
1985	13	15	-2	-1	2	4
	75	**80**	**0**	**0**	**14**	**10**

Find Alpha.
Answer :

$\overline{X}=15$; $\overline{Y}= 16$; $x = X - \overline{X}$; $y = Y - \overline{Y}$

$$\text{Covariance between X \& Y} = \frac{\sum xy}{n} = \frac{14}{5} = 2.80$$

$$\text{Variance of market portfolio} = \frac{\sum x^2}{n} = \frac{10}{5} = 2$$

$$\text{Beta} = \frac{2.80}{2} = 1.40.$$

Alpha = 16 – (1.40 x 15) = -5

(Note : This security is 1.40 times riskier as compared to the market. If the return from the market goes down to 0 i.e. market return fall by 15 percentage points, return from this security will fall by 15x1.40 i.e. 21 percentage points. At present, the likely return from the security is 16%; when it will fall by 21 percentage points it will go down to -5)

Q.No.56: Risk free rate of return is 8%.The return from market portfolio is expected to be 12%. Using the date given below, find the security which is better investment. Also find Alpha of each security.

Security	Expected return	Beta
A	15	1.00
B	14	1.50

Answer

The expected rate of return given in the question is actually 'Likely return']

SECURITY	LIKELY RETURN	EXPECTED RETURN
A	15	8 + 1(12-8) = 12
B	14	8 + 1.5(12-8) = 14

Security A is better than security B, as its likely return is more than what the investor expects (considering the risk the investor has undertaken).

ALPHA = Likely Return – (Beta x RM)
Alpha of A = 15 – (1 x 12) = 3
Alpha of B = 14 – (1.5 x 12) = – 4

Q. No. 57 : From the following data, find Alpha, Beta and Systematic risk of equity shares of Babbar Ltd.

Period	% Return of equity shares of Babbar Ltd.	% Return of market portfolio
1	30	35
2	28	30
3	25	18
4	20	15
5	24	20
6	10	11
7	3	-10

Answer :

Let return of Babbar Ltd = X, return of Market = Y

X	Y	x	y	x^2	y^2	xy
30	35	10	18	100	324	180
28	30	8	13	64	169	104

25	18	5	1	25	1	5
20	15	0	-2	0	4	0
24	20	4	3	16	9	12
10	11	-10	-6	100	36	60
3	-10	-17	-27	289	729	459
				594	1272	820

Mean of X = 140/7 =20 Mean of Y = 119/7 = 17

SD OF X = 9.21 SD OF Y = 13.48

Covariance = $\sum xy/n$ = 820/7 = 117.1428

Beta = covariance / market variance = $(117.1428)/(13.48)^2$ = 0.645

Alpha = likely return – (Beta x RM) = 20 – (0.645x17) = 9.035

Systematic risk = $Beta^2$ x market variance = $(0.645)^2 \times (13.48)^2$ = 75.60

<u>**CHARACTERISTIC LINE**</u>: A characteristic line exhibits regression relationship between the return of an investment and the return on market portfolio.
Equation of Characteristic Line = Alpha + (Beta x RM)
(Return from investment)

Q. No.58 : The rates of return on the security of company Y and market portfolio for 10 periods are given below:

Period	Return from security Y %	Return on market portfolio %
1	20	22
2	22	20
3	25	18
4	21	16
5	18	20
6	-5	8
7	17	-6
8	19	5
9	-7	6
10	20	11

What is Beta? What is Alpha? What is the Characteristic line of the security? Draw Characteristic line. (**Nov. 2003**)

Answer

(X = return on market portfolio; Y = return on Y security)

Period	X	Y	x	y	xy	x^2
1	22	20	10	5	50	100
2	20	22	8	7	56	64
3	18	25	6	10	60	36
4	16	21	4	6	24	16
5	20	18	8	3	24	64
6	8	-5	-4	-20	80	16
7	-6	17	-18	2	-36	324
8	5	19	-7	4	-28	49
9	6	-7	-6	-22	132	36
10	11	20	-1	5	-5	1
Total	120	150	0	0	357	706

Answer

$\overline{X}=12$; $\overline{Y}= 15$; $x = X - \overline{X}$; $y = Y - \overline{Y}$

$$\text{Covariance between X \& Y} = \frac{\sum xy}{n} = \frac{357}{10} = 35.70$$

$$\text{Variance of market portfolio} = \frac{\sum x^2}{n} = \frac{706}{10} = 70.60$$

$$\text{Beta} = \frac{35.70}{70.60} = 0.5057$$

Alpha = Average return of the security – (Beta x RM)

15 – (0.5057 x 12) = 8.9316

Characteristic Line of security = Alpha + (Beta x RM)

(Return from investment)

= 8.9316 + (0.5057 x RM)

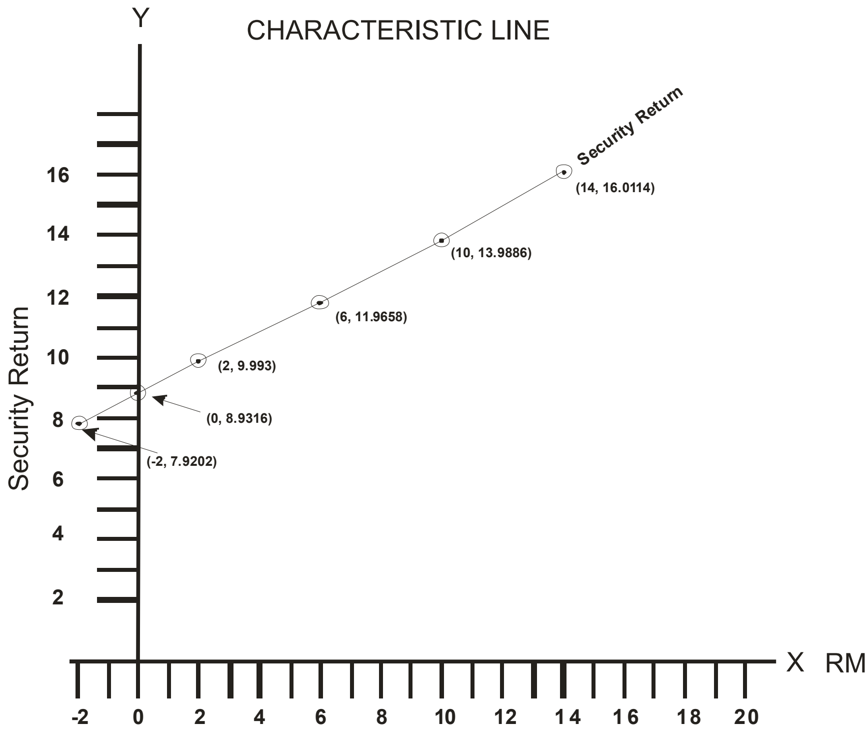

Diagram 10

Teaching note – not to be given in the exam.

(i) There are two equations of regression (a statistical technique); one is X on Y and the other is Y on X. From the X on Y, we can find the values of X given the values of Y and not vice-versa. Similarly, from Y on X, we can find the values of Y, given the values of X and not vice-versa.

While defining the term characteristic line, we have used the term regression. This term has been used to state point out that this equation can be used only for finding the Return from Investment (security) given RM and not vice-versa.

(ii) The term line means an equation that states the relationship between certain variables. For example, here we establish relationship between return from security and return from market portfolio.

If we are not asked to draw the line, it means that we simply have to state the equation.

If we are asked to draw the line, we should exhibit this relationship by way of line on the graph. For this purpose, we may take any 2 (minimum) values of RM, calculate the corresponding values of the return from the security and plot these values to draw a line on the graph.

To draw this line, RM is taken on X-axis and return on a particular investment is taken on Y-axis.

SECURITY MARKET LINE (SML)

A security market line exhibits relationship between expected returns (calculated on the basis of CAPM) of investments and their Betas. (By expected return we mean, the total return an investor should get considering the risk he has undertaken)

To draw the line, Betas are taken on X-axis and the expected returns on Y-axis.

Q. No.59 : RF 10%. RM 15%, From the following information, draw SML:

Securities	Likely Return	Beta
E.Shares of A Ltd.	13.00%	0.50
E.Shares of B Ltd.	14.00%	1.00
E.Shares of C Ltd.	18.00%	1.50
E.Shares of D Ltd.	20.00%	2.00

Which share(s) should be undervalued / overvalued?

> **Teaching note:** ["Undervalued" means good investments. These are the investments regarding which our recommendation is : purchase / hold. Overvalued means "not good investments". These are the investments regarding which our recommendation is : sale.]

Answer

SECURITY	LIKELY RETURN	EXPECTED RETURN	COMMENT
A	13%	10+ 0.50(15-10) =12.50	UNERVALUED
B	14%	10+ 1.00(15-10)=15.00	OVERVALUED
C	18%	10+ 1.50(15-10)=17.50	UNDERVALUED
D	20%	10+ 2.00(15-10)=20.00	Neutral

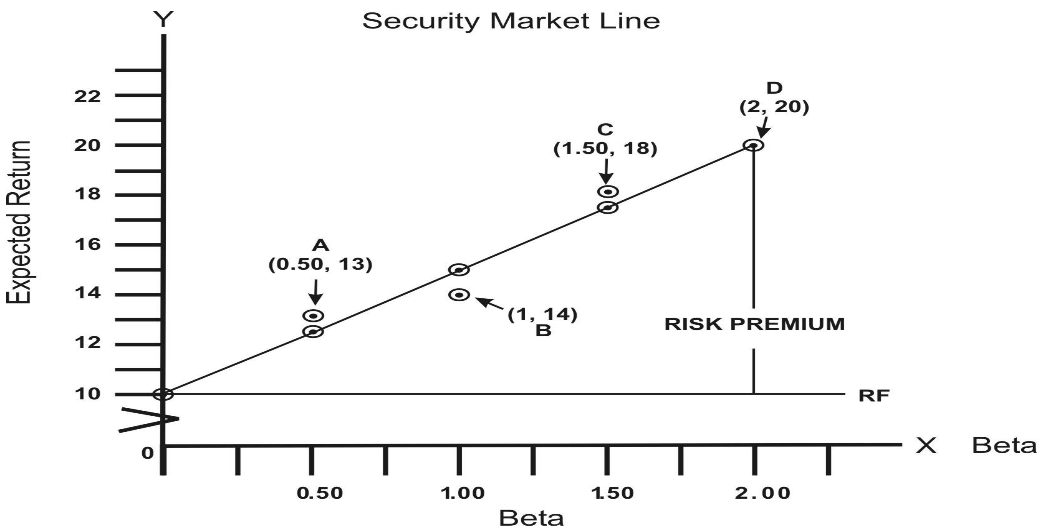

Diagram 11

CAPITAL MARKET LINE (CML)

A CML exhibits relationship between expected returns of investments and their standard-deviations. (By expected return we mean, the total return an investor should get considering the risk he has undertaken). To draw this line, SDs are taken on X-axis and the expected returns on Y-axis.

Q. No. 60:The following data relate to four different portfolios:

Portfolio	Expected Rate of Return	S.D. of Returns from Portfolios
A	16%	6.00
B	14%	7.50
C	12%	3.00
D	15%	9.00

The expected return on market portfolio is 9.50 percent with a standard deviation of 3. The RF is 5 per cent. Draw CML to comment on each of these portfolios.

Answer

Portfolio	LIKELY RETURN	EXPECTED RETURN	COMMENT
A	16%	5+ 9.00 = 14	UNERVALUED
B	14%	5+ 11.25 = 16.25	OVERVALUED
C	12%	5+ 4.50 = 9.50	UNDERVALUED
D	15%	5+ 13.50 = 18.50	OVERVALUED

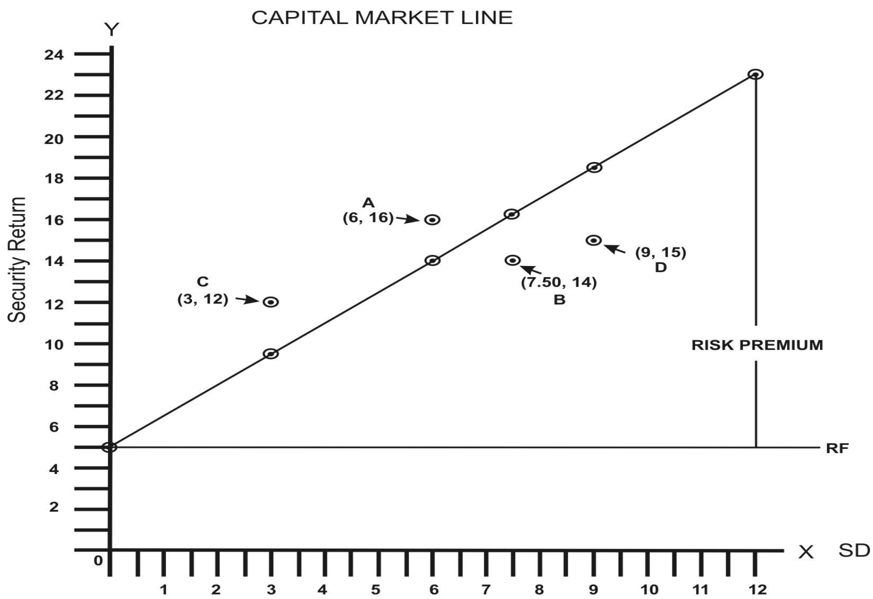

Diagram 12

MARKET MODEL

CAMP states the expected return of an investment. (By expected return we mean, the total return an investor should get considering the risk he has undertaken). Market Model (MM) states the likely return of an investment in the form of following equation:

Likely Return or return based on MM:

Risk adjusted Excess Return + RF + Beta (RM-RF)

Example

Alpha 4%, RF 6%, Beta = 2
RM 10%, Find return based on MM.

Answer :

If return from market portfolio is zero, i.e. return from market portfolio falls by 10 percentage points, the return from security will fall by 20%; even than the return from security will be 4%. It means likely return from the security is 24%. Expected return from the security is : 6 + 2 (10-6) = 14%. Hence, risk adjusted excess return = 10%. (This risk adjusted excess return is also called as Jenson's Alpha. It is different from the Alpha that we have studied in the first paragraph of this note)

Likely Return i.e. return based on MM: 10 + 6 + 2 (10-6) = 24%

MATRIX APPROACH IN INVESTMENT DECISIONS

Investment consultants favour the matrix approach because of its clear focus of the factors affecting the investment decisions.. A matrix is formed of various acceptable investment opportunities and factors affecting the investment decisions. Three important decisions affecting the factors are : (i) Return (ii) Risk and (iii) Liquidity. Acceptable standard levels are decided. Each acceptable level is considered worth 100 points. The points allotted for return changes in direct proportion of the likely returns from the investments under our consideration. The points allotted for risk and liquidity changes in reverse proportion to the standards for acceptable levels.

Q. No.61: Suppose an investor expects a return of 15%, his acceptable level of risk is 4 and he expects a liquidity of 2 days. He gathers the following likely data for four investments. Recommend one security for investment following the Matrix Approach.

	RETURN	RISK	LIQUIDITY
A	12%	2	2 DAYS
B	18%	4	1 DAY
C	24%	8	4 DAYS
D	15%	1	8 DAYS

Answer
Investment Matrix

	Return	Risk	Liquidity	Total
A	80	200	100	380
B	120	100	200	420
C	160	50	50	260
D	100	400	25	525

As per the matrix, D is recommended for the investment.

GENERAL PROBLEMS

Q. No. 62 : The data for three securities P,Q, and R are as follows:

Securities	Likely return	SD
P	15%	0.20
Q	15%	0.19
R	20%	0.24

Does anyone security dominates another? Why? Which type of investor prefers R?

Teaching note : Security A dominates Security B in following three situations:
- (i) A and B have same returns but A's SD is less than that of B.
- (ii) A's return is higher than that of B but their standard deviations are equal.
- (iii) A's return is higher than that of B but A's SD is less than that of B

Answer
Q dominates P as Q's return is the same as that of P but its (Q's) SD is lower than that of P. An aggressive type investor prefers R . Aggressive investors invest in securities with higher expected returns and for this they undertake higher degree of risk.

Q. No. 63: 4 D plc is manufacturing company whose share are listed on the London International Stock Exchange. It is expecting to have surplus cash resources available for at least 12 months. The board has decided to develop an investment portfolio of marketable securities. The company's financial advisers have recommended four securities for the board to consider. These are as follows:

Security 1 regularly traded shares in a medium-sized UK retailing company. The equity beta is quoted as 1.2.

Security 2 shares in a relatively small but rapidly growing UK company in a high-technology industry. The shares have an equity beta of 1.6.

Security 3 shares in an American bank which are listed on US stock exchanges but not in the UK. They are currently quoted at US $ 25.50. An equity beta is unavailable, but 4 D plc's stockbroker estimates that the expected rate of return on the shares is 12% per annum.

Security 4 short-dated government bonds.

The expected return on Treasury Bills is 5% per year, and that of the market is 12%. 4 D plc's equity beta is 0.8 and this is not expected to change in the foreseeable future.

The board can invest in one or more of these securities in any proportion.

Calculate the risk and expected return of the investment portfolio, assuming 30% of available funds is invested in each of securities 1 and 2 and 20% in each of securities 3 and 4.

Answer

Risk of portfolio: [(0.30x1.20)+ (0.30x1.60)+ (0.20x1)+ (0.20x0)]=1.04

Return of portfolio:

[(0.30x13.40)+ (0.30x16.20)+ (0.20x12)+ (0.20x5)]= 12.28

Q. No. 64 : An investor has two portfolios known to be on minimum variance set for a population of three securities A, B and C having below mentioned weights:

	WA	WB	WC
Portfolio X	0.30	0.40	0.30
Portfolio Y	0.20	0.50	0.30

It is supposed that there are no restrictions on short sales.

(i) What would be the weight for each stock for a portfolio constructed by investing Rs. 5,000 in portfolio X and Rs. 3,000 in portfolio Y?.

(ii) Suppose the investor invests Rs. 4,000 out of Rs. 8,000 in security A. How he will allocate the balance between security B and C to ensure that his portfolio is on minimum variance set? (6 Marks) (**June 2009 SFM**)

Answer :

(i) Weight for each stock for the portfolio :

	Investment	
Security A	5000 x 0.30 + 3000 x 0.20 = 2,100	2,100/8,000 = 0.2625
Security B	5000 x 0.40 + 3000 x 0.50 = 3,500	3,500/8,000 = 0.4375
Security C	5000 x 0.30 + 3000 x 0.30 = 2,400	2,400/8,000 = 0.3000

(ii) Both the Portfolios X and Y are minimum variance portfolios. In both of these portfolios, the weight of C is 0.30. We interpret this information that for minimum variance portfolio, the investment in C should be 8000 x 0.30 i.e. Rs. 2400. Balance of Rs. 1600 should be invested in B.

Q. No.65: Shyam Sunder has been holding 1,00,000 equity shares of Mohan Ltd costing Rs.20,00,000. He borrowed 50,000 equity shares of Gopal Ltd. and sold these shares at the rate of Rs.20. He invested the sale proceeds of these shares in the shares of Mohan Ltd. Expected return of equity shares of Mohan Ltd is 12 % and that of Gopal Ltd is 18 %.Coefficient of correlation between the return of the two securities is 0.70. SD of returns of the two companies are: Mohan 0.20 and Gopal 0.80. Find the expected return and risk of Shyam Sunder's investments.

Answer

W_1 = 1.50 W_2 = −0.50

Expected Return of investment = (1.50)(12) + (−0.50)(18) = 9

Portfolio SD =

$$\sqrt{(1.50)^2(0.20)^2 + (-0.50)^2(0.80)^2 + 2(1.50)(-0.50)(0.70)(0.20)(0.80)} = 0.29$$

The investor's decision is not optimum as this has reduced the expected income and increased the risk.

Q. No.66: The following date relates to a portfolio :

Year	Nominal return (%)	Inflation (%)
2001	20	5
2002	30	10
2003	40	7
2004	5	6
2005	-2	2

Find the SD of the real returns in different years.

Answer:

(1+Real Return)(1+Inflation) = 1 + Nominal return

- (1+ Real Return) (1.05) = 1.20

Real return in 2001 = 0.1429 = 14.29%

- (1+ Real Return) (1.10) = 1.30

Real return in 2002 = 18.18%

- (1+ Real Return) (1.07) = 1.40

Real return in 2003 = 30.84

- (1+ Real Return) (1.06) = 1.05

Real return in 2004 = − 0.9433 %

- (1+ Real Return) (1.02) = 0.98

Real return in 2005 = − 3.92 %

X	x	x^2
14.29	2.60	6.76
18.18	6.49	42.12
30.84	19.15	366.72
- 0.9433	-12.63	159.17
- 3.92	-15.61	243.67
		818.44

Mean = 11.69

SD = 12.79

Q. No.67: SD of market returns is 25%. (a) Estimate the SD of portfolio with a Beta of 1.20. Assume that the portfolio funds have been allocated in about 50 sectors of the economy. (b) Estimate the SD of a portfolio which has all the investment in Government securities. (c) Estimate the Beta of a well-diversified portfolio having S.D. 10 %. (d) SD of a portfolio is 25%. The portfolio consists shares of only 3 sectors of the economy i.e. Power, Pharma and IT. Can you say some thing about its Beta?

Answer

(a) This a well diversified portfolio. In such portfolio, the unsystematic risk is almost nil. In the market portfolio also, the unsystematic risk is nil. SD of market represents systematic risk. Beta of 1.20 indicates that the portfolio is 1.20 times riskier than that of market. Hence SD of the portfolio is 30%.

(b) Zero

(c) In a well diversified portfolio, the unsystematic risk is almost nil. In the market portfolio also, the unsystematic risk is nil. SD of market represents systematic risk. SD of this portfolio represents the systematic risk of this portfolio. Its SD is only 40 % of the market SD. In other words this portfolio is 0.40 times riskier than the market. Market Beta is 1. Hence the Beta of the portfolio is 0.40.

(d) SD of the market is 25% (it has only systematic risk). SD of the portfolio (not well diversified) is 25% (it has systematic as well as unsystematic risk). In means in this portfolio the systematic risk is less than 25%. It means its Beta is less than 1.

Q. No. 68: A portfolio consists of equity shares of 20 companies, with equal amount of investment in all the 20 companies. The Beta of shares of 12 companies if 1.20 and that of 8 companies is 0.70. Find the Beta of the total Investment.

Answer

60% of the investment is in 12 companies having a Beta of 1.20 and 40% of the investment is in 8 companies having a Beta of 0.70.

Portfolio beta = 0.60x 1.20 + 0.40x 0.70 = 1.00

Q. No.69: How many variance items and how many covariance items you require to calculate the risk of a portfolio consisting of 40 different securities.

Answer
No. of variance items = n = 40
No. of covariance items = ${}^{n}C_2$ = 780

Q. No. 70: A portfolio consists of 25 different securities, with equal amount of investment in all the 25 Securities. SD of each security is 0.60. Coefficient of correlation with each other's return is 0.40. What is the SD of the portfolio?
Answer

Portfolio SD= $\sqrt{25.(0.60)^2(0.04)^2 + 300.2.(0.04)^2.(0.40)(0.60)^2}$ = 0.391

Q. No. 71: Expected dividend per share Rs.8. Ke = 10%. g = 5%. Find the value of the share. If Expected EPS is Rs.12, find the rate of return on retained earnings. What portion of the market price is for growth opportunities (i.e. for a ROE on retained earnings over and above Ke)?

Answer

$$P = \frac{D_1}{Ke - g} = \frac{8}{0.10 - 0.05} = 160$$

g = b.r Where g = growth rate, B= retained EPS to EPS
r = rate of return on retained earnings
0.05 = (1/3)r
r = 15 %

If r would have been 10% : g = b.r = (1/3)(0.10) = 0.0333

$$P = \frac{D_1}{Ke - g} = \frac{7.8728}{0.10 - 0.0333} = 118.09$$

Market price for growth opportunities = 41.91

Q. No. 72. BSE Index 5,000.
Value of portfolio Rs.10,10,000
Risk free rate : 9% p.a.
Dividend yield on Index : 6% p.a.
Beta of portfolio : 1.50

We assume that a future contract on the BSE index for four months is used to hedge the value of the portfolio over next three months. One futures contract is for delivery of 50 times the index.
Based on the above information calculate:
(i) Price of futures contract
(ii) The gain on short future position if the index turns out to be 4,500 in three months. **(Nov. 2007)**

Answer

(i) Price of the futures contract: Spot price + carrying cost – returns

= 5000 + 5000 x(0.09 x 4/12) – 5000 x(0.06 x 4/12)

= 5050

(ii) Value of one futures contract = 5050x50 = 2,52,500

No. of contracts for hedging = (1.50x10,10,000) / 252500 = 6

Sell 6 futures contract maturity 4 months.

Price of futures contract (maturity one month):

4500 + 4500 x(0.09 x 1/12) – 4500 x(0.06 x 1/12)

= 4511.25

Close the futures contract @ Rs.4511.25

Profit on futures = 50 x 6 x (5050 – 4511.25) = Rs.1,61,625

Q. No. 73: Last year the dividend per share was Rs.3. Dividend has been growing at the rate of 5 per annum and this growth is likely to be maintained. Find the current price of the share assuming the required rate of return to be 10% p.a. for first 5 years, 12% p.a. for next 5 years and 15% p.a. after that for infinite period.

Answer

Year	Dividend per share
1	3.1500
2	3.3075
3	3.4729
4	3.6465
5	3.8289
6	4.0203
7	4.2213
8	4.4324
9	4.6540
10	4.8867
11	5.1310
12	5.3876

P = [3.15/(1.10)] + [3.3075/(1.10)^2] + [3.4729/(1.10)^3] [3.6465/(1.10)^4] [3.8289/(1.10)^5] + [4.0203/{(1.10)^5.(1.12)}] + [4.2213/{(1.10)^5.(1.12)^2}] + ……………………………+ [4.8867/{(1.10)^5.(1.12)^5}] + [{4.8867(1.05)}/{(1.10)^5.(1.12)^5.(1.15)}]+ [{4.8867(1.05)^2}/{(1.10)^5.(1.12)^5.(1.15)^2}]+ …………… ∞

= [3.15/(1.10)] + [3.3075/(1.10)^2] + [3.4729/(1.10)^3] [3.6465/(1.10)^4] [3.8289/(1.10)^5] + [4.0203/{(1.10)^5.(1.12)}] + [4.2213/{(1.10)^5.(1.12)^2}] + ……………………………+ [4.8867/{(1.10)^5.(1.12)^5}] + [{4.8867(1.05)}/{(1.10)^5.(1.12)^5.(1.15)}] + [{4.8867(1.05)^2}/{(1.10)^5.(1.12)^5.(1.15)^2}]+ …………… ∞

$$= 22.91 + \frac{[\{4.8867(1.05)\}/\{(1.10)^5.(1.12)^5.(1.15)\}]}{1 - [1.05 / 1.15]} = 22.91 + 18.07 = 40.98$$

Q. No. 74: An investor has invested Rs. 1,00,000 in five different securities. He is interested in investing another Rs. 20,000. He is considering the following securities:

	BETA
A	1.00
B	0.50
C	0
D	– 0.50
E	– 0. 70

Suggest the security in which he should invest assuming that his object is the risk reduction.

Answer For maximum risk reduction, the investor should invest in E.

Q. No. 75 : Expected returns on two stocks for particular market returns are given below :

Market return	Aggressive	Defensive
7%	4%	9%
25%	40%	18%

You are required to calculate: (a) the betas of the two stocks

(b) Expected return of each stock if the market return is equally likely to be 7% or 25%

(c) The Security market line if the RF is 7.50% and the market return is equally likely to be 7% or 25%.

(d) The Alphas of the two stocks. **(May, 2007)**

Answer:

(a) Aggressive : When market return increased from 7% to 25% i.e. by 18%, the return of aggressive security increased from 4% to 40% i.e. by 36 %. It means the Beta is 2.

Defensive : When market return increased from 7% to 25 % i.e. by 18%, the return of defensive increased from 9% to 18% i.e. by 9%. It means the Beta is 0.50.

(b) Expected return of Aggressive = 4(0.50) + 40 (0.50) = 22

Expected return of defensive = 9(0.50) + 18(0.50) = 13.50

(c) Expected return of market portfolio : 7(0.50) + 25 (0.50) = 16%

Security market line = RF + Beta (RM-RF) = 7.50 + Beta (16 -7.50)
= 7.50 + 8.50Beta

(d) Alpha = Return from security – Beta x RM

Alpha of Aggressive = 22 – 2(16) = -10

Alpha of defensive = 13.50 – 0.50(16) = 5.50

Q. No.76 A study by a mutual fund has revealed the following data in respect of three securities:

Security	SD(%)	Correlation with Index. Pm[15]
A	20	0.60
B	18	0.95
C	12	0.75

The SD of the market portfolio has been observed as 15%.
(i) What is the sensitivity of returns of each stock with respect to the market?
(ii) What are the co-variances among the various stock?
(iii) What would be the risk of the portfolio consisting of all the stocks equally?
(iv) What is the Beta of the portfolio consisting of equal investment in each stock?
(v) What is the total systematic and unsystematic risk of the portfolio in (iv).
(Nov. 2009)

Answer

(i) Security A :

$$\text{Coefficient of correlation} = \frac{\text{Covariance}}{SD_A \times SD_{Market}}$$

$$0.60 = \frac{\text{Covariance}}{0.20 \times 0.15}$$

Covariance = 0.018

Security B :

$$\text{Coefficient of correlation} = \frac{\text{Covariance}}{SD_B \times SD_{Market}}$$

$$0.95 = \frac{\text{Covariance}}{0.18 \times 0.15}$$

Covariance = 0.02565

[15] Portfolio market

Security C:

$$\text{Coefficient of correlation} = \frac{\text{Covariance}}{SD_C \times SD_{Market}}$$

$$0.75 = \frac{\text{Covariance}}{0.12 \times 0.15}$$

Covariance = 0.0135

Beta (sensitivity) of A = $0.018/(0.15)^2$ = 0.80

Beta (sensitivity) of B = $0.02565/(0.15)^2$ = 1.14

Beta (sensitivity) of C = $0.0135 / (0.15)^2$ = 0.6

(ii) Assumption : Let the coefficient correlation between A & B =1
Let the coefficient correlation between A & C =1
Let the coefficient correlation between B &C =1

A & B

$$\text{Coefficient of correlation} = \frac{\text{Covariance}}{SD_A \times SD_B}$$

$$1 = \frac{\text{Covariance}}{0.20 \times 0.18}$$

Covariance = 0.036

A & C

$$\text{Coefficient of correlation} = \frac{\text{Covariance}}{SD_A \times SD_C}$$

$$1 = \frac{\text{Covariance}}{0.20 \times 0.12}$$

Covariance = 0.024

B & C

$$\text{Coefficient of correlation} = \frac{\text{Covariance}}{SD_B \times SD_C}$$

$$1 = \frac{\text{Covariance}}{0.18 \times 0.12}$$

Covariance = 0.0216

(iii) Variance of portfolio = $(W_1SD_A)^2 + (W_2SD_B)^2 + (W_3SD_C)^2 + 2W_1W_2rSD_ASD_B + 2W_1W_3rSD_ASD_c + 2W_2W_3rSD_BSD_C$

$= (0.3333x0.20)^2 + (0.3333x0.18)^2 + (0.3333x0.12)^2$
$+ 2x0.3333x0.3333x1x0.20x0.18$
$+ 2x0.3333x0.3333x1x0.20x0.12$
$+ 2x0.3333x0.3333x1x0.18x0.12$
$= 0.00444 + 0.00360 + 0.001560$
$+ 0.0080 + 0.0053 + 0.0048 = 0.0277$

SD of portfolio = $\sqrt{0.0277}$ = 0.1664

(ii) Beta of portfolio = (0.80 + 1.14 + 0.60)/ 3 = 0.8467

(iii) Systematic Risk of portfolio (variance) = $(0.8467)^2(0.15)^2$ = 0.016

Total risk of portfolio (variance) = 0.0277

Unsystematic risk(variance) = 0.0277 – 0.016 = 0.0117

INTERNATIONAL INVESTING

The world is getting smaller every day. As economies grow more interconnected, investment in cross border investments in shares and debt instruments is multiplying. International investing has two key advantages (i) higher return and strong diversification (which reduces the risk).

Higher Return:

(i) Economic conditions: Economic conditions are not the same in all the countries. At a particular time, some economies grow must faster than other economies. For example, currently the economies of BRIC are growing a much faster rate than the economies of USA and Europe. Higher rate of growth of the economies lead to faster growth of the companies of those economies. International investing targets at multiplying the wealth of the investors through investing in these companies.

(ii) Foreign exchange expectations: Some times international investing is done to take the advantage of expected appreciation of the currency of the target country. Many FIIs have taken this advantage by investing in India. Suppose an FII invested in India when 1$ = Rs.45. They take back their investment proceeds when 1$ = Rs.40.

Diversification:

Stock market volatility is a major reason for investing globally. Investing in several different countries can help reduce the impact on your portfolio when one region experiences an economic downturn. This variety in market performance can work to smooth your total portfolio performance through the power of diversification. International investing faces many risks like political risk, information risk, high transactions cost risk, foreign exchange risk etc. The political risk cannot be quantified, it can only be analyzed. Hence, finance mangers do not have much role to play while considering this risk. Information risk has been reduced recently transparency in financial markets is increasing throughout the globe. The transaction cost is pre-determined and this risk is known at the time of investment. Hence, it not poses any threat.

The main risk is Foreign exchange risk. Suppose an FII invests in a country. After some time the FII wants to withdraw from that country. If during the period of investment, the currency of the country (in which investment was made) depreciates, the FII suffers loss; even the gain made on the investment may turn into loss.

For example, A Swiss FI invested 1m CHFs in India when the foreign exchange rate was 1 CHF = Rs.30. The investment value grows by 10%. Now the FII wants to withdraw from India. Suppose, the foreign exchange rate is 1 CHF = Rs.35. The amount to be repatriated is Rs.33m/35 i.e 0.9429m CHFs. i.e. negative return of 5.708 %.

International investing approach differs from Internal investing mainly on account of foreign exchange risk. While calculating the expected return and risk of the cross border investing, the investor has to consider the foreign exchange risk.

$Rii = (1 + Ri)(1 + Rfe) - 1$

Rii = Return from international investing
Ri = Return from investment in terms of currency in which investment has been made.

Rfe = % decline or appreciation in the currency of the country in which investment has been made.

In the above example, $Ri = 0.10$

% discount of Rupee $= \{(1/35) - (1/30)] / [(1/30)] = 0.1428$

$Rfe = -0.1428$

$Rii = (1 + Ri)(1+Rfe) - 1 = (1 + 0.10)(1 - 0.1428) - 1 = -0.05708$
$= -5.708\%$

SD of Returns from international investing :

$SDii = \sqrt{(SDi)^2 + (SDfe)^2 + 2.r.(i,fe)(SDi)(SDfe)}$

Q.No.77: Risk free rate of return in China is 12%. An Austrian company is considering the investment in some Chinese securities. These securities have a Beta of 1.48 and the variance of their returns is 20%. RM in China is 22%. The Chinese Yuan is likely to deprecate by 5.56% annually against the Euro with a variance of 15%. Coefficient of correlation between the returns from investment in these securities and those from foreign exchange fluctuations is 0.21. Find the expected annual return and variance of the investment of the Austrian investor.

Answer :
Expected return in China = RF + Beta (RM-RF) = 12 + 1.48(22-12)
= 26.80%
Rii = (1 + Ri)(1+Rfe) – 1
= (1+ 0.268)(1 - 0.0556) – 1 = 19.75%

SD of Returns from international investing :

$$SDii = \sqrt{(SDi)^2 + (SDfe)^2 + 2.r.(i,fe)(SDi)(SDfe)}$$

$$SDii = \sqrt{(0.20) + (0.15) + 2(0.21)(0.4472)(0.3873)} = 0.6502$$

Variance = 0.4227 = 42.27%

Q. No. 78: A French Investor invested 1m Euros in Indian stock market when the FER was 1 Euro = Rs.55.50. The investment appreciated by 8% in terms of rupees during the year. The year end FER is 1 Euro = Rs.57.67. What is the rate of return of the French investor?

Answer :
Rate of return = wealth ratio (in investor's home currency) - 1
= [55.50(1.08)/57.67]/1 - 1 = 3.94%

Q. No. 79: Suppose a US investor wishes to invest in a British firm equity share currently selling for £ 100 per share. The investor considering an investment of $ 10,000 when the exchange rate is 1£ = 2$.

(a) Fill in the following table for the rates of return after 1 year in each of nine scenarios:

Price per share (£)	Pound denominated return (%)	Dollar denominated Return if year end exchange rate is :		
		$ 1.88 /£	$2/£	$2.12/£
95	-5%	A	B	C
100	0	D	E	F
105	+5%	G	H	I

(b) When is the Dollar denominated return equal to the Pound denominated return?

(c) Find the expected Pound denominated return and SD of Pound denominated returns. Assume equal probability for each of the three probable prices after 1 year.

(d) If the outcome of each of 9 outcomes is equally likely, find (i) the Expected Dollar denominated return and (ii) SD of Dollar denominated returns.

(e) Suppose the investor sells £ 5,100 on forward basis (Maturity: 1 year) at the rate of $ 2.07 /£ : find (i) the Expected Dollar denominated return and (ii) SD of Dollar denominated returns. Assume equal probability for each possible outcome.

(f) Compare the SD calculated under (c) and (d)

(g) Compare the SD calculated under (d) and (e)

Answer (a)

Price per share (£) at year end	Pound denominated return (%)	Dollar denominated Return if year end exchange rate is :		
		$ 1.88 /£	$2/£	$2.12/£
95	-5%	A	B	C
100	0	D	E	F
105	+5%	G	H	I

The investor purchased 50 shares @ £100 each

A	[(50x 95x1.88) / 10,000] – 1	-10.70%
B	[(50x95x 2.00) / 10,000] – 1	- 5.00%
C	[(50x95x 2.12) / 10,000] – 1	+0.70%
D	[(50x 100x1.88) /10,000] – 1	-6.00%
E	[(50x100x 2.00) /10,000] – 1	0%
F	[(50x100x 2.12) /10,000] – 1	+6.00%
G	[(50x 105x1.88) /10,000] – 1	-1.30%
H	[(50x105x 2.00) /10,000] – 1	+5.00%
I	[(50x105x 2.12) /10,000] – 1	+11.30%

Answer (b)

Dollar denominated return equals to Pound denominated return when there is no change in Foreign exchange rate, i.e. year end foreign exchange rate = beginning of the year rate : 1 Pound = 2 $

Answer (c) X = Pound denominated return

X	Probability(p)	pX	x	px^2
-5	1/3	-5/3	-5	8.33
0	1/3	0	0	0
+5	1/3	+5/3	-5	8.33
		$\sum pX = 0$		$\sum px^2 = 16.66$

Expected return = 0 SD = $\sqrt{16.66}$ = 4.08

Answer (d) X = $ denominated return

X	Probability(p)	pX	x	px^2
−10.70%	1/9	−10.70/9 =−1.19	−10.70	12.72
− 5.00%	1/9	−0.56	− 5.00	2.78
+ 0.70%	1/9	+ 0.07	+ 0.70	0.054
−6.00%	1/9	−0.67	−6.00	4.00
0%	1/9	0	0	0
+ 6.00%	1/9	+ 0.67	+ 6.00	4
−1.30%	1/9	−0.14	−1.30	0.188
+ 5.00%	1/9	+ 0.56	+ 5.00	2.78
+ 11.30%	1/9	+ 1.26	+ 11.30	14.188
		$\sum pX$ = 0		$\sum px^2$ = 40.71

Expected return = 0 SD = $\sqrt{40.71}$ = 6.38

Answer (e)

A	[(50x 95x1.88) + (5100 x 0.19)] / 10,000 − 1	−1.01%
B	[(50x95x 2.00) + (5100 x 0.07)] / 10,000 − 1	−1.43%
C	[{50x95x 2.12}+ {(5100) x (−0.05)}] / 10,000 − 1	−1.85%
D	[(50x 100x1.88) + (5100 x 0.19) /10,000] − 1	+ 3.69%
E	[(50x100x 2.00) + (5100 x 0.07)/10,000] − 1	+ 3.57%
F	[{50x100x 2.12}+ {(5100) x (−0.05)}] /10,000 − 1	+ 3.45%
G	[(50x 105x1.88) + (5100 x 0.19)] /10,000] − 1	+ 8.39%
H	[(50x105x 2.00) + (5100 x 0.07) /10,000] − 1	+ 8.57%
I	[{50x105x 2.12)}+ {(5100) x (−0.05)}]/10,000] − 1	+ 8.75%

X = Dollar denominated return with forward

X	Probability(p)	pX	x	px^2
−1.01%	1/9	−0.1122	−4.58	2.3307
−1.43%	1/9	−0.1589	−5.00	2.7778
−1.85%	1/9	−0.2055	−5.42	3.2640
+ 3.69%	1/9	+ 0.4100	+ 0.12	0.0016
+ 3.57%	1/9	+ 0.3966	0	0
+ 3.45%	1/9	+ 0.3833	−0.12	0.0016
+ 8.39%	1/9	+ 0.9322	+ 4.82	2.5814
+ 8.57%	1/9	+ 0.9522	+ 5.00	2.7778
+ 8.75%	1/9	+ 0.9722	+ 5.18	2.9814
		$\sum pX$ = 3.5699		$\sum px^2$ =16.7163

Expected return = 3.5699 SD = $\sqrt{16.7163}$ = 4.09

Answer (f)

SD under (c) : 4.08

SD under (d) : 6.38

The SD under (d) is more than that under (c) as while (c) has only investment risk, (d) has not only investment risk but also foreign exchange risk.

Answer (g)

SD under (d) : 6.38

SD under (e) : 4.09

Both the cases are based on $ denominated returns. SD under (e) is less as compared to (d) as under (d) the risk has been reduced with the help of forward contract.

Q. No. 80 : If the current exchange rate is Euro 1.50/£, one year forward contract rate is Euro 1.56/£, the interest rate in a British Government security is 7%. Find risk free rate of return for the French investor if the investment is made for one year.

Answer :

Let the investor brings Euro 1,500 for investment. This is equal to £ 1,000.

At the end of the year, this will grow to £1,070.

Rate of return = wealth ratio (in investor's home currency) – 1

= [(1070x1.56)/1500] – 1 = 11.28%

Q. No. 81 : You are a Chinese investor considering the purchase of one of the following securities. (Face value of the Chinese Government Security is CY 100 and that of the French Government security is Euro 100)

Bond	Time horizon	Coupon	Price
French Govt.	6 months	7.50%	100
Chinese Govt.	6 months	6.50%	100

Calculate the expected % change in FE rate which would result in the two bonds having equal total return in CY over 6 months time horizon. Verify your calculations assuming the Chinese investor invested CY 1,00,000 and the foreign exchange rate at the time of investment was : 1 Euro = 5 CY

Answer

Investment on Chinese Government security = 3.25%

Return on French Government security (In CY) = (1.0375)(1-RFE) – 1

For equal return : 0.0325 = (1.0375)(1-RFE) – 1

RFE = 0.4819% (discount on Euro)

Verification

Spot rate : 1 Euro = 5CY

Expected rate after 6 months : 1 Euro = [(5) –(5x0.4819/100)]CY

= 4.9759 CY

Return on French Govt security (in CY) = (1.0375)(1-0.004819) – 1 = 0.0325

= 3.25%

Q. No. 82: You are an Italian investor who purchased Canadian securities for CD 1,950 1 year ago when the foreign exchange rate was: 1 Euro = 1.50 CD. The value of the security, now, is CD 2,340 and the exchange rate is: 1 Euro = 1.80 CD. Find the rate of return of the investor.

Answer

Return = Wealth ratio (in Euro) -1

= [(2340/1.80) /(1950/1.50)] -1 = 0

Q. No. 83: An Investment manager of a Luxemburg Mutual Fund plans to invest Euro 15m in domestic securities for 60 days. The investment policy of the mutual fund permits her to invest abroad provided the foreign exchange risk is covered through forward contract.

What rate of return will the manager earn if she invests either in Chine or Australia and hedges the Euro value of her investment proceeds through forward? Use the data given in the following table.

What must be the approximate 60 days interest rate in Euro denominated Government securities.

Interest rates: (60 days)

China government 6 % p.a.

Australian government 3 % p.a.

Exchange rate EURO per unit of foreign currency

	Spot	60 days forward
Chinese Yuan	0.2000	0.2010
Australian Dollar	0.5000	0.5025

Assume 360 days in a year.

Answer : 60 days interest rate in Euro denominated Government securities

Spot rate : 1 CY = 0.2000 Euro

60 days forward rate : 1 CY(1.01) = 0.2000Euro(1+r)

(where r is interest on one Euro for two months)

1 CY = 0.1980 Euro (1+r)

We are given : 1 CY = 0.2010

0.2010 = 0.1980Euro (1+r)

Euro (1+r) = 0.2010/0.1980 =1.01515

r= 0.01515 = 1.515% = 9.09% p.a.

Alternative solution : 1 AD = 0.50 Euro
60 days forward rate : 1 AD(1.005) = 0.5000Euro(1+r)
(where r is interest on one Euro for two months)
1 AD = 0.4975 Euro (1+r)
We are given : 1 AD = 0.5025Euro
0.5025 Euro = 0.49751244Euro (1+r)
(1+r) = 0.5025/0.49751244 =1.01025
r= 0.01025 = 1.0025% = 6.015% p.a.

The forward rates are not based on IRPT, otherwise by both the methods the interest of Euro would have been same. Hence, Euro interest rate can not be determined.

Return (Euro) = Wealth ratio in Euro -1

Investment in China : Return (Euro) = [75m(1.01)x(0.2010)]/(15m)]-1
= 0.01505% (for two months)
Investment in Australia: Return (Euro)= [30m(1.005)x(0.5025)]/(15m)]-1
= 0.0025% (for two months)

Q. No. 84 : A Chinese investor invests CY 1,00,000 in an Australian government security carrying 8% p.a. interest when the foreign exchange rate was 1AD = 4CY.

(a) What is the CY denominated return for one year if the year end exchange rate is CY 3.50, CY 4.00 or CY 4.50? Assume no hedging of the foreign exchange risk.

(b) What is the CY denominated return for one year if the investor covered his foreign exchange risk by booking the forward contract (one year maturity). Assume CY denominated securities yield at 6% p.a. and that the forward rates are based on IRPT.

(c) What is the CY denominated return for one year if the investor covered his foreign exchange risk by booking the forward contract (one year maturity) at CY 4.12?

Answer

(a) At the time of investment; CY 1,00,000 = 25,000 Australian Dollars

Return (CY) = Wealth ratio in CY -1

	Year end exchange rate		
Return (in CY)	1AD = 3.5CY	1AD = 4CY	1AD = 4.50CY
Wealth at the time of investment	1,00,000 CY	1,00,000 CY	1,00,000 CY
Wealth at year end	25000(1.08)x3.50 = 94,500 CY	25000(1.08)(4) = 1,08,000 CY	125000(1.08)(4.50) = 1,21,500 CY
Return	(94500/100000) - 1 = -5.50%	(108000/100000) -1 = 8%	(121500/100000) -1 = 21.50%

(b) Spot rate : 1 AD = 4 C

Forward rate : 1 AD(1.08) = 4 CY (1.06)

: 1 AD = 3.9259

Return (CY) = Wealth ratio in CY –1
= [(25000x1.08) x 3.9259]/(100000) –1 = 6%

(c) Return (CY) = Wealth ratio in CY –1
= [(25000x1.08) x 4.12]/(100000) –1 = 11.24%

Q. No. 85: Hari, a Canadian investor, is considering investing 40% of his investing funds in domestic market and balance either in Britain or France. Coefficient of correlation between returns on the shares between Canada and France is 0.50.

Coefficient of correlation between returns on the shares between Canada and Britain is 0.30. Expected return in each country is 15%. Variance of returns in each country is 16%? Advise whether France or Britain?

Answer

Expected return is same. Variance of returns is same. The decision of the investment would be guided by only one factor, i.e. risk reduction through portfolio. For this purpose, we prefer the portfolio of the securities having least coefficient of correlation.

In this case, Hari should invest in Canada and Britain.

There will be no reduction in his income but the risk would be reduced.

Q. No. 86: A French Investor invested 1m Euros in Indian stock market when the FER was 1 Euro = Rs.55. The investment appreciated by 10% in terms of rupees during the year. The end FER is 1 Euro = Rs.57. What is the rate of return of the French investor?

Answer

Suppose the investor invested 1 Euro.
Investment in Rupee = Rs.55
Maturity value = Rs.60.50
No. of Euros = 60.50/57 = 1.0614
Return = Wealth ratio –1 = [(1.0614/1) – 1] = 0.0614 = 6.14%

Q. No. 87: Consider following annual rates of return and foreign exchange rates:

Year	Rates of return in		Beginning of the year exchange rate of Australian Dollar per Euro
	Australian Dollar	French Euro	
2001	12%	7%	A$2.50
2002	- 7%	15%	A$2.00
2003	11%	- 5%	A$1.75
2004	10%	9%	A$2.00
2005	7%	11%	A$1.60
2006			A$1.50

Determine the Simple average rate of return of an Australian investor who invests in France.
Determine the Simple average rate of return of a French investor who invests in Australia.

Answer
Australian Investor :

Return on investment	Return on foreign currency	Return in home currency
7%	% gain/loss on Euro =(2-2.50)/2.50 = -20%	(1.07)(1-.20) -1 = - 14.40%
15%	% gain/loss on Euro =(1.75-2.00)/2.00 = -12.50%	(1.15)(1-.125) -1 = 0.625%
-5%	% gain/loss on Euro =(2.00-1.75)/1.75 = 14.2857%	(1.142857)(1-.05) - 1 = 8.57%
9%	% gain/loss on Euro =(1.60-2.00)/2.00 = -20%	(1.09)(1-.20) - 1 = -12.80%
11%	% gain/loss on Euro =(1.50-1.60)/1.60 = -6.25%	(1.11)(1-.0625) - 1 = 4.0625
Total		-13.9425
Simple Average		- 2.7885%

French Investor

Return on investment	Return on foreign currency	Return in home currency
12%	% gain/loss on A$ =(0.50-0.40)/0.40 = 25%	(1.12)(1.25) -1 = 40%
- 7%	% gain/loss on A$ =(0.571-0.50)/0.50 = 14.20%	(1-.07)(1.1420) -1 = 6.206%
11%	% gain/loss on A$ = (0.50-0.5714)/0.5714 = -12.50%	(1.11)(1-.125) -1 = -2.875%
10%	% gain/loss on A$ =(0.625-0.50)/0.50 = 25%	(1.10)(1.25) - 1 = 37.50%
7%	% gain/loss on A$ = (0.66667-0.625)/0.625 = 6.67%	(1.07)(1.0667) - 1 = 14.14%
Total		94.791
Simple Average		18.96%

ARBITRAGE PRICING THEORY (APT)

The APT is based upon the assumption that there are a few major macro-economic factors that influence security returns. In other words the APT holds that

(i) the expected return from a security is a linear function of various factors affecting the returns from the securities in the market; these factors may be interest rate, inflation, currency rates, GDP growth, oil prices etc., each factor is represented by a factor specific beta coefficient.

(ii) the correct price of the asset can be determined using the APT model derived expected rate.

This method is an alternative to Capital Asset Pricing Model which assumes that the expected return is linearly related to only one specific factor and that is market return. We know that this market return is affected by various macro factors[16] like interest rate, inflation, GDP growth, currency rates, oil prices etc. (Macro factors are also referred as risk factors.)

[16] Macro factors are also referred as risk factors.

The CAPM considers the effect of these factors in totality i.e. in the form of market return; the APT considers these factors separately.

As per APT: Expected return
= RF + [$Beta_1$ x Δ Risk $Factor_1$]+ [$Beta_2$ x Δ Risk $factor_2$]+ …………………...+
[$Beta_n$ x Δ Risk $factor_n$]

This can also be presented as follows:

As per APT: Expected return = $\lambda_{0} + (\lambda_{1}.b_1 + \lambda_{2}.b_2 + \lambda_{3}.b_3 ... + \lambda_{n}.b_n)$

There are two parts of expected return.
(i) the first part is λ_0 ; (it is pronounced as Lambda zero) ; it represents risk free return. In APT, the term RF return refers to the expected rate of return if the macro-factors (which affect the return from the security) remain unchanged. Suppose the return from a particular share is affected by four macro-factors (a) GDP growth (b) Inflation (c) interest rates and (d) oil prices. The expectation is that GDP will grow at 5%, inflation will be at 3%, interest rates will be at 7% and oil prices will be at $ 60. If all this happens, the share is likely to have a return of 10%. In the above formula, RF refers to this rate i.e. RF is taken as 10%.

(ii) the remaining part represents risk premium. The Lambda parameters (i.e. λ_1 , λ_2 , λ_3 , …….. λ_n) represent change in the units of the macro-factors that affect the risk premium; the examples of such factors are interest rates, oil prices, GDP growth, inflation rates etc. b_1 ,.b_2 ,b_3……b_n are referred as Betas or sensitivity factors. A Beta factor represents change in risk premium in case of one unit change of a macro-factor affecting the risk premium.

Beta value of any factor is taken as one if the security has average exposure to that factor.

Units of macro-factors are of two types:

(a) Macro-factors are given in percentage: In this case the change in the units of macro-factors is measured in terms of percentage points. Suppose RF was calculated assuming that GDP growth will be 5%; the revised estimates are that GDP will grow at 7%, the change in the units of GDP growth will be taken as 2%. If λ_1 represents change in GDP growth, the value of λ_1 is 2%.

(b) Macro-factors are given in absolute figures. In this case the change in the units of macro-factors is measured in terms of terms of change in absolute figures

Suppose the return from a particular share is affected by four macro-factors (a) GDP growth (b) Inflation (c) interest rates and (d) oil prices. Assuming that GDP will grow at 5%, inflation will be at 3%, interest rates will be at 7% and oil prices will be at $ 60, the RF was calculated as 10%. The revised estimates are that GDP will grow at 9%, inflation will be at 2%, interest rates will be at 8% and oil prices will move from $ 60 to $ 66. The sensitivity factors for GDP growth inflation, interest rates and oil prices are 0.10, -0.10, -0.12 and -0.03 respectively.

Expected return = $\lambda_{0} + (\lambda_{1}.b_1 + \lambda_{2}.b_2 + \lambda_{3}.b_3 ... + \lambda_{n}.b_n)$

= 10 + [{(3) x (0.10)} + {(-1) x (-0.10)} + {(1) x (-0.12)} + {(6) x (-0.03)}]
= 10.10 %

In some exam questions, we are given Risk Free rate (risk free rate in the real sense; for example treasury bill rate or Government security interest rate). In such questions, we consider the absolute units of macro factors and not change in units of macro-factors. Example: Suppose the treasury bill rate is 8%. The return from a share depends upon 3 factors (a) GDP Growth (b) Inflation and (c) interest rates. The Betas of GDP growth, inflation and interest rates are 0.50, -0.05 and -0.01 respectively. Find the expected return if GDP grows by 8%, inflation is 2% and interest is 5%. In this case, the expected return is:
8 + {8 x 0.50} + {(2x)-(0.05)} + {(5)x(-0.01)} = 11.85 %

Q . No. 88: RF rate : 7%. Using the following data, find the expected return of a portfolio assuming that 40% of the funds are invested in security A and 60% in B.

Macro-factors	λ	Beta (A)	Beta (B)
1	8 %	0.60	0.90
2	5 %	1.10	0.80
3	- 4 %	0.90	1.20

Answer : Expected return of A = $\lambda_0 + (\lambda_{1}.b_1 + \lambda_{2}.b_2 + \lambda_{3}.b_3)$
= 0.07+ [{(0.08) x (0.60)}+ {(0.05) x (1.10)} + {(-0.04)x (0.90)}]
= 0.137 = 13.70 %

Expected return of B = $\lambda_0 + (\lambda_{1}.b_1 + \lambda_{2}.b_2 + \lambda_{3}.b_3)$
= 0.07+ [{(0.08)x(0.90)}+ {(0.05)x(0.80)}+ {(-0.04)x(1.20)}]
= 0.134 = 13.40%

Expected return of the portfolio : 0.40 (13.70) + 0.60 (13.40) = 13.52 %

ARBITRAGE

Arbitrage is the process of taking advantage of the price differential. Arbitrage actions by different arbitrageurs tend to remove the price differential. Arbitrage is possible in the following three cases :

(i) The same asset quotes at different prices in different markets. For example, prices of shares of a particular company in BSE and NSE

(ii) There are different expected returns for two or more similar assets (i.e. the assets having the same risk). For example, suppose there are two assets having the same risk, theoretically, their expected returns must be same. If their returns are different, the arbitrageur will sell the assets having lower return and buy the assets having the higher returns and have some profit.

(iii) The realization from an asset on a future date is certain. If today its price is not equal to the present value of the amount to be realized, the present value should be calculated using risk free rate.

The APT, developed by Stephen Ross in 1976, deals with the second possibility of arbitrage. If there are two assets which have same risk, theoretically their expected returns should be same. If their expected returns are different, the arbitrageurs will sell the assets having lower returns) and buy the assets having higher returns and have some profit.

The following discussion will help us in understanding the arbitrage process.

Q No.89: Suppose there is only one macro factor in an economy and that is growth rate of GDP. For share A, the Beta of this factor is 1.19 (i.e. if GDP growth increases by 1 percentage point, the value of the share increase by 1.19 percentage point and vice versa.) The GDP is expected to grow at 5.50% and based on this the required return of the share is 9.50%. Suppose the government announces that GDP is expected to grow by 6.30%, what will be the required rate of return?

Answer Required return of A = $\lambda_0 + (\lambda_{1.}b_1)$

$= 9.50 + (0.80 \text{x} 1.19) = 10.452\%$

Q. No. 90: Suppose there is only one macro factor in an economy and that is growth rate of GDP. For share A, the Beta of this factor is 1.18 (i.e. if GDP growth increases by 1 percentage point, the value of the share increase by 1.18 percentage point and vice versa.) For share B, the Beta of this factor is 0.76.Required return of A is 13% and that of B 9.50%. What is λ_0?

Answer :

Required return $= \lambda_0 + (\lambda_{1.}b_1)$

Required return of A : $13.00 = \lambda_0 + ((\lambda_{1.}\text{X } 1.18)$

Required return of B : $9.50 = \lambda_0 + (\lambda_{1.}\text{X } 0.76)$

Solving the equations, we get $\lambda_0 = 3.16$

Q. No. 91: Suppose that there are only two macro factors in an economy-Industrial production (IP) and inflation rate (IR). IP is expected to change by 3%

and IR 5%. A stock has beta of 1 on the IP and -0.50 on IR. Currently the required rate of return is 12%. What will be the required rate of return if the expectation of IP changes to 6% and that of inflation to 9%?

Answer

Expected return $= \lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2)$

$= 12 + (3 \times 1) + \{(4) \times (-0.50)\} = 13\%$

Q. No. 92: The expected return on equity shares of A Ltd is affected by two factors i.e. inflation rate and interest rate. The change is inflation rate is represented by λ_1 and that of interest rate by λ_2. The return is described as follows:

Expected return $= \lambda_0 + (0.5\lambda_1 + 1.2\lambda_2)$

Assuming λ_0 is 8%, λ_1 is 5.50% and λ_2 is 2.30%, determine the expected return.

Answer

Expected return $= \lambda_0 + (0.5\lambda_1 + 1.2\lambda_2)$

$= 8 + (0.50 \times 5.50) + (1.20 \times 2.30) = 13.51\%$

Q. No. 93 : The expected return on equity shares of A Ltd. and B Ltd is affected by three factors i.e. factor 1, factor 2 and factor 3. Factor 1 is represented by λ_1, factor 2 by λ_2 and factor 3 by λ_3. Assuming the RF to be 10.50%, determine the required returns of the two companies using the following data :

Factor	λ	Beta (A Ltd)	Beta(B Ltd)
1	10 %	0.41	0.62
2	15%	0.62	0.42
3	- 2%	0.13	0.06

Answer

Expected return $= \lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2 + \lambda_{23}b_3)$

Expected return of A $= 10.50 + (10 \times 0.41 + 15 \times 0.62) - (2 \times 0.13) = 23.64$

Expected return of B $= 10.50 + (10 \times 0..62 + 15 \times 0.42) - (2 \times 0.06) = 22.68$

Q. No. 94: The expected return on equity shares of A Ltd., B Ltd and C Ltd is affected by two factors. Assuming the RF to be 10.45%, determine the required returns of the three companies using the following data :

Factor	λ	Beta (A)	Beta (B)	Beta (C)
1	11%	0.71	0.22	1.13
2	5%	0.19	0.19	0.34

Answer : Expected return $= \lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2)$

Expected return of A $= 10.45 + (11 \times 0.71 + 5 \times 0.19) = 19.21$

Expected return 0f B $= 10.45 + (11 \times 0.22 + 5 \times 0.19) = 13.82$

Expected return 0f C $= 10.45 + (11 \times 1.13 + 5 \times 0.34) = 24.58$

Q. No. 95 : The expected return on equity shares of A Ltd is affected by three factors i.e. factor 1, factor 2 and factor 3. Factor 1 is represented by λ_1, factor 2 by λ_2 and factor 3 by λ_3. Assume the RF is 10%.

An investor is interested in purchasing one equity share of this company. He estimates that he shall be able to sell it for Rs.501.50 after holding the same for one year. Determine the present worth of this share, using the following details :

Factor	λ	Beta
1	15%	0.49
2	15%	0.58
3	- 0.02%	0.12

Answer

Expected return of A = $\lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2 + \lambda_3.b_3)$

= 10 + (15 x 0.49) + (15 x 0.58) – (0.02 x 0.12) = 26.0476

Present worth of share = 501.50/1.260476 = 397.47

Q. No. 96 : The earnings and dividends of X Ltd. have been growing at the rate of 8% for past so many years and the trend is likely to continue in foreseeable future. The company has just paid a dividend of Rs. 5 per share. RF is 8%. Determine the present worth of this share, using the following details :

Factor	λ	Beta
1	15%	0.54
2	15%	0.67
3	- 0.02%	0.11

Answer

Expected return of X = $\lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2 + \lambda_3.b_3)$

= 8 + (15 x 0.54) + (15 x 0.67) – (0.02 x 0.11) = 26.1478

Present worth of share = D_1 /(Ke – g) = 5.40 / (0.261478 - 0.08) = 29.76

Q No. 97: The probability distribution of the likely annual return on Equity shares of Y Ltd is given below :

Likely return	Probability
10%	0.16
12%	0.24
15%	0.35
18%	0.15
20%	0.10

The required return on the equity shares is determined using the following equation: 7% + 1.70%.b_1 + 2.10%.b_2 + 0.66%.b_3. Assuming b_1, b_2 and b_3 be 0.28, 0.44 and -0.22, advise whether investment should be made in these equity shares or not.

Answer : Expected return of Y= λ_0 + ($\lambda_1.b_1$ + $\lambda_2.b_2$+ $\lambda_3.b_3$)
= 7 + (1.70 x 0.28) + (2.10 x 0.44) – (0.66 x 0.22)
= 8.25

Likely return = 10x0.16 + 12x0.24 + 15x0.35 + 18x0.15 + 20x0.10
= 14.43

As the likely return is more than the required return, the investment is recommended.

Q. No.98: Assume that there is only one factor that affects the return of the equity shares. Given the following data of two shares, determine the Equilibrium Arbitrage Pricing line:

Share	Expected Return	Beta
A	16%	1.60
B	10%	0.60

Teaching note: Equilibrium Equation or equilibrium model (also referred as Equilibrium Arbitrage Pricing Line): This is an equation which is consistent with the data of given two or more portfolios.

Answer : Expected return = λ_0 + $\lambda_1.b_1$

$16 = \lambda_0 + \lambda_1.(1.60)$ ………..(1)

$10 = \lambda_0 + \lambda_1.(0.60)$ ………..(2)

Subtracting (2) from (1) :

$\lambda_1 = 6$

Putting this value in (1) or (2) : λ_0 = 6.40

Equilibrium Arbitrage pricing line = λ_0 + $\lambda_1.b_1$= 6.40 + 6.b_1

Q. No.99: Assume the following equilibrium – priced assets exist :

Portfolio	Expected return	b_1	b_2
A	16.10%	0.90	0.75
B	12.80%	0.66	0.58
C	18.50%	0.96	1.18

What is the equilibrium equation of two factor APT model?

Answer

$16.10 = \lambda_0 + (\lambda_1 \text{ x } 0.90 + \lambda_2 \text{ x } 0.75)$

$12.80 = \lambda_0 + (\lambda_1 \text{ x } 0.66 + \lambda_2 \text{ x } 0.58)$

$18.50 = \lambda_0 + (\lambda_1 \text{ x } 0.96 + \lambda_2 \text{ x } 1.18)$

Solving the equations, we get : λ_0 = 3.272
λ_1 = 10.87
λ_2 = 4.06

Equilibrium equation = 3.272 + 10.87b_1 + 4.06b_2

Q.No.100 : Mr A has invested Rs.2,00,000 in the following Portfolio :

	Expected Return	Beta	Weights
Share of A	30%	0.46	0.50
Share of B	18%	1.37	0.40
Share of C	17%	0.58	0.10

He has been advised to change the weights by 0.05, 0.025 and -0.075. Advise.

Answer

Present :

Expected portfolio return = 30 x 0.50 + 18 x 0.40 + 17 x 0.10 = 23.90%

Portfolio Beta = 0.46 x 0.50 + 1.37 x 0.40 + 0.58 x 0.10 = 0.836

Changed scenario:

Expected portfolio return =30 x 0.55 + 18 x 0.425 + 17 x 0.025 = 24.575%

Portfolio Beta =0.46 x 0.55 + 1.37 x 0.425 + 0.58 x 0.025 = 0.84975

The investment is recommended provided the investor is will to bear extra risk.

Q. No. 101: Assume that there is only one factor that affects the return of the equity shares. Given the following data of two shares, determine the Equilibrium Arbitrage Pricing line:

Share	Expected Return	Beta
A	16%	1.10
B	12%	0.70

Answer : Expected return = $\lambda_0 + \lambda_1.b_1$

$16 = \lambda_0 + \lambda_1.(1.10)$ ………..(1)

$12 = \lambda_0 + \lambda_1.(0.70)$ ………..(2) :

Solving the equations:

$\lambda_{1.} = 10$ %

Putting this value in (1) or (2) : $\lambda_0 = 5\%$

Equilibrium Arbitrage pricing line = $\lambda_0 + \lambda_1.b_1 = 5 + 10.b_1$

Q.No.102: Continuing with Q. No. 14, Assume that another security (C), with following details, is available in the market:

Expected Return = 15% Beta = 1.05

Is there some arbitrage opportunity? Apply APT.

Answer: As per APT, we have to compare the assets having same risk (Beta). Beta of C is 1.05 while that of A and B is 1.10 and 0.70 respectively. To make these Betas comparable, let's constitute a portfolio of A and B which will have Beta of 1.05.

$1.05 = [(W_1)(1.10) + (1-W_1)(0.70)]$

Solving the equation: $W_1 = 0.875$ $1 - W_1 = W_2 = 0.125$

Expected return of this portfolio: (0.875x 16) + (0.125x12) = 15.50

Asset	Expected return	Beta
C	15%	1.05
Portfolio of A & B with W_1 as 0.875 and W_2 as 0.125	15.50	1.05

Both the assets have same Beta but C's return is lower. Hence arbitrage profit is possible through Shorting (Borrow the securities and selling them) C and purchasing A & B.

Borrow shares of C having current market value of Rs.1,00,000. Sell these shares. Use the sale proceeds to buy shares of A Rs.87500 and shares of B Rs.12500.

Asset	Initial cash flow	Year end cash flows
Portfolio of A & B with W_1 as 0.875 and W_2 as 0.125	- 100000	+ 1,15,500
C	+ 100000	-115000
	0	+ 500

Q No. 103 : Consider the following data for one factor (GDP) economy.

Portfolio	Expected return	Beta
A	11%	1.18
B	6%	0

Suppose there is another portfolio C with Beta of 0.59 and expected return of 8%.What would be the arbitrage opportunity?

Answer:

As per APT, we have to compare the assets having same risk (Beta).
Beta of C is 0.59 while that of A and B is 1.18 and 0 respectively. To make these Betas comparable, let's constitute a portfolio of A & B which will have Beta 0.59.

$0.59 = [(W_1)(1.18) + (1-W_1)(0)]$

Solving the equation: $W_1 = 0.50$ $1 - W_1 = W_2 = 0.50$

Expected return of this portfolio: (0.50x 11) + (0.50x6) = 8.50

Asset	Expected return	Beta
C	8.00%	0.59
Portfolio of A & B with W_1 as 0.50 and W_2 as 0.50	8.50%	0.59

Both the assets have same Beta but C's return is lower. Hence arbitrage profit is possible through Shorting (Borrow the securities and selling them) C and purchasing A & B.

Borrow shares of C having current market value of Rs.1,00,000. Sell these shares. Use the sale proceeds to buy shares of A Rs.50000 and shares of B Rs.50000.

Asset	Initial cash flow	Year end cash flows
Portfolio of A & B with W_1 as 0.50 and W_2 as 0.50	- 100000	+ 1,08,000
C	+ 100000	-1,08,500
	0	+ 500

Q. No. 104: Using the following details, can we make arbitrage profit :

	Expected Return	Beta $_1$	Beta $_2$
Share of A	17%	1.10	0.70
Share of B	11%	0.50	0.60
Share of C	16.50%	0.80	0.65

Answer : As per APT, we have to compare the assets having same risk (Beta).

$Beta_1$ of C is 0.80 while that of A and B is 1.10 and 0.50 respectively. To make these Betas comparable, let's constitute a portfolio of A and B which will have Beta of 0.80.

$0.80 = [(W_1)(1.10) + (1-W_1)(0.50)]$

Solving the equation: $W_1 = 0.50$ $1 - W_1 = W_2 = 0.50$

$Beta_2$ of C is 0.65 while that of A and B is 0.70 and 0.60 respectively. To make these Betas comparable, let's constitute a portfolio of A and B which will have Beta of 0.65.

$0.65 = [(W_1)(0.70) + (1-W_1)(0.60)]$

Solving the equation: $W_1 = 0.50$ $1 - W_1 = W_2 = 0.50$

Expected return of this portfolio: (0.50x 17) + (0.50x11) = 14

Asset	Expected return	$Beta_1$	$Beta_2$	
C	16.50%	0.80	0.65	
Portfolio of A & B with W_1 as 0.50 and W_2 as 0.50	14%	0.80	0.65	

Both the assets have same Beta but C's return is higher. Hence arbitrage profit is possible through shorting (Borrow the securities and selling them) A & B and purchasing C.

Borrow shares of A & having current market value of Rs.50,000 each. Sell these shares. Use the sale proceeds to buy shares of C Rs.100000.

Asset	Initial cash flow	Year end cash flows
Portfolio of A & B with W_1 as 0.50 and W_2 as 0.50	+ 100000	−1,14,000
C	−100000	+ 1,16500
	0	+ 2500

Q. No. 105 The expected return on equity shares of A Ltd is affected by three macro factors i.e. Change in GDP Growth, change Oil prices and change in Monsoon. Assume the RF is 10%. The values of these factors are as follows:

Macro Factors :	Value of Macro factor
Change in GDP	6%
Change in Oil prices	−2%
Change in Monsoon	3%

(i) Suppose a share has average exposure to each of these factors. Expected Return?

(ii) A share has very high exposure to Change in GDP growth (Beta = 2.50) but no exposure to other factors. Expected return?

(iii) A share has very high exposure to Change in oil prices (Beta = 3), average exposure to change in GDP but no exposure to change in Monsoon. Expected return?

(iv) A share has no exposure to all these factors. Expected return?

Answer

(i) The term 'average exposure' is used to denote that beta of that macro factor is one.

Expected return = $\lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2 + \lambda_3.b_3)$

$= 10 + (6x1) + (-2x1) + (3\ x1) = 17\%$

(ii) Expected return = $\lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2 + \lambda_3.b_3)$

$= 10 + (6 \ x\ 2.50) + (-2\ x\ 0) + (3\ x\ 0) = 25\%$

(iii) Expected return = $\lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2 + \lambda_3.b_3)$

$= 10 + (6\ x\ 1) + (-2\ x\ 3) + (3\ x\ 0) = 10\%$

(iv) Expected return = $\lambda_0 + (\lambda_1.b_1 + \lambda_2.b_2 + \lambda_3.b_3)$

$= 10 + (6\ x\ 0) + (-2\ x\ 0) + (3\ x\ 0) = 10\%$

Q. No. 106 The expected return on equity shares in an economy is affected by four macro factors i.e. Change in GDP Growth, change Oil prices, change in Monsoon and change in Book value. Assume the RF is 10%. The values of these factors are as follows:

Macro Factors :	Value of Macro factor
Change in GDP	6%
Change in Oil prices	-2%
Change in Monsoon	3%
Change in Book value	10%

Given the following factor sensitivities of the equity shares of four companies, find the expected return on equity shares of each company.

	Factor sensitivities			
Factor	Madhav Ltd.	Keshav Ltd.	Krishna Ltd.	Damodar Ltd.
Change in GDP	1.50	2.00	0.50	0.20
Change in Oil prices	-1.00	-0.05	-0.10	-0.90
Change in Monsoon	0.20	0.50	2.00	1.50
Change in Book value	0.25	0.50	0.65	0.55

Answer :

Expected return = λ_0 + ($\lambda_{1.}b_1$ + $\lambda_{2.}b_2$+ $\lambda_{3.}b_3$ + $\lambda_{34}b_4$)

Expected return of Madhav:10+ (6x1.50)+ (-2 x -1)+ (3 x0.20)+ (10 x 0.25)
= 24.10 %

Expected return of Keshav:10+ (6x 2)+ (-2 x -0.05)+ (3 x0.50)+ (10 x 0.50)
= 28.40 %

Expected return of Krisjhna:10+ (6x0.50)+ (-2 x -0.10)+ (3 x 2.00)+ (10 x 0.65)
= 25.30%

Expected return of Damodar:10+ (6x0.20)+ (-2 x -0.90)+ (3 x1.50)+ (10 x 0.55)=
= 19.40% %

Q. No.107 The expected return on equity shares in an economy is affected by four macro factors i.e. Change in GDP Growth, change Oil prices, change in Monsoon and change in Book value. Assume the RF is 10%. The values of these factors are as follows:

Macro Factors :	Value of Macro factor
Change in GDP	6%
Change in Oil prices	-2%
Change in Monsoon	3%
Change in Book value	10%

Given the following factor sensitivities of the equity shares of four companies, find the expected return on equity shares of each company.

	Factor sensitivities			
Factor	Madhav Ltd.	Keshav Ltd.	Krishna Ltd.	Damodar Ltd.
Change in GDP	1.50	2.00	0.50	0.20
Change in Oil prices	-1.00	-0.05	-0.10	-0.90
Change in Monsoon	0.20	0.50	2.00	1.50
Change in Book value	0.25	0.50	0.65	0.55

Murari had Rs.1,00,000 to invest. He borrowed 100 shares of Damodar Ltd. and sold at the rate of Rs.500. He invested Rs. 150000 equally in the equity shares of other three companies. Find the Expected return of the portfolio.

Answer:

W_1 = 50,000 / (50,000 + 50,000 + 50,000 – 50,000) = 0.50

W_2 = 50,000 / (50,000 + 50,000 + 50,000 – 50,000) = 0.50

W_3 = 50,000 / (50,000 + 50,000 + 50,000 – 50,000) =0.50

W_4 = -50,000/ (50,000 + 50,000 + 50,000 – 50,000) =-0.50

Beta of portfolio (change in GDP) =

= 1.50 x 0.50 + 2.00 x 0.50 + 0.50 x 0.50 – 0.20 x 0.50

= 1.90

Beta of portfolio (change in Oil prices) =

= – 1.00 x 0.50 – 0.05 x 0.50 - 0.10 x 0.50 + 0.90 x 0.50

= -0.125

Beta of portfolio (change in Monsoon) =

= 0.20 x 0.50 + 0.50 x 0.50 + 2.00 x 0.50 – 1.50 x 0.50

= 0.60

Beta of portfolio (change in Book value) =

= 0.25 x 0.50 + 0.50 x 0.50 + 0.65 x 0.50 - 0.55 x 0.50

= 0.425

Expected return of the portfolio = λ_0 + ($\lambda_{1.}b_1$ + $\lambda_{2.}b_2$+ $\lambda_{3.}b_3$ + $\lambda_{34}b_4$)

= 10 + (6.00 x 1.90) + (-2)x(-0.125) + 3x0.60+ 10x0.425

= 27.70%

Note : The commission on borrowing the shares has not been considered as it is not given in the question.

Q. No. 108 The expected return on equity shares in an economy is affected by four macro factors i.e. Change in GDP Growth, change Oil prices, change in Monsoon and change in Book value. Assume the RF is 10%. The risk premiums of these factors are as follows:

Macro Factors :	Factor Risk premium
Change in GDP	6%
Change in Oil prices	-2%
Change in Monsoon	3%
Change in Book value	10%

Given the following factor sensitivities of the equity shares of four companies, find the expected return on equity shares of each company.

	Factor sensitivities			
Factor	Madhav Ltd.	Keshav Ltd.	Krishna Ltd.	Damodar Ltd.
Change in GDP	1.50	2.00	0.50	0.20
Change in Oil prices	-1.00	-0.05	-0.10	-0.90
Change in Monsoon	0.20	0.50	2.00	1.50
Change in Book value	0.25	0.50	0.65	0.55

Murari had Rs.1,00,000 to invest. Girdhari Borrowed 400 shares of Damodar and 250 shares of Krishna, he sold these shares at the rate Rs.500 and Rs. 400 respectively. He invested Rs. 4,00,000 equally in other two companies. Find the Expected return of the portfolio.

Answer ;

[Teaching note- not to be given in exam.
We are given only one series of factor risk premium. There are four companies - so we should have been four series of risk premium. In this case the given factor risk premium is considered as risk premium for average exposure i.e. the given risk premium for Beta = 1. In other words we can say that the given factor risk premium denotes macro factors , also called as risk factors.

W_1 = 2,00,000 / (2,00,000 + 2,00,000 – 1,00,000 – 2,00,000) = 2
W_2 = 2,00,000 / (2,00,000 + 2,00,000 – 1,00,000 – 2,00,000) = 2
W_3 = -1,00,000 / (2,00,000 + 2,00,000 – 1,00,000 – 2,00,000) = -1
W_4 = -2,00,000 / (2,00,000 + 2,00,000 – 1,00,000 – 2,00,000) = -2

Beta of portfolio (change in GDP) =
= 1.50 x 2 + 2.00 x 2 + 0.50 x (-1) + 0.20 x (-2)
= 6.10

Beta of portfolio (change in Oil prices) =
= - 1.00 x 2 – 0.05 x 2 - 0.10 x (-1) - 0.90 x (-2) = -0.20

Beta of portfolio (change in Monsoon) =
= 0.20 x 2 + 0.50 x 2 + 2.00 x (-1) + 1.50 x (-2)
= - 5.60

Beta of portfolio (change in Book value) =
= 0.25 x 2 + 0.50 x 2 + 0.65 x (-1) + 0.55 x (-2)
= - 0.25

Expected return of the portfolio = $\lambda_0 + (\lambda_{1.}b_1 + \lambda_{2.}b_2 + \lambda_{3.}b_3 + \lambda_{34}b_4)$
= 10 + (6.00 x 6.10) + (-2)x(-0.20) + 3x(-5.60)+ 10x(-0.25)
= 27.20%

Note : The commission on borrowing the shares has not been considered as it is not given in the question.

Q. No. 109 The expected return on equity shares in an economy is affected by four macro factors i.e. Change in GDP Growth, change Oil prices, change in Monsoon and change in Book value. Assume the return from Treasury Bills is 10%.

Given the following data, find the expected return as well actual return on equity shares of each company.

	Factor sensitivities			
Factor	Madhav Ltd.	Keshav Ltd.	Krishna Ltd.	Damodar Ltd.
Change in GDP	1.50	2.00	0.50	0.20
Change in Oil prices	-1.00	-0.05	-0.10	-0.90
Change in Monsoon	0.20	0.50	2.00	1.50
Change in Book value	0.25	0.50	0.65	0.55

Factor	Expected rate of change	Actual rate of change
Change in GDP	6%	5%
Change in Oil prices	2%	3%
Change in Monsoon	3%	2%
Change in Book value	5%	6%

Answer :

Expected return = $\lambda_0 + (\lambda_{1.}b_1 + \lambda_{2.}b_2 + \lambda_{3.}b_3 + \lambda_{34}b_4)$

Expected return of Madhav;10+ {6x1.50}+ {2x(-1)}+ {3x0.20}+ {5x0.25}
=18.85%

Expected return of Keshav;10+ {6x2}+ {2x(-0.05)}+ {3x0.50}+ {5x0.50}
=25.90%

Expected return of Krishna;10+ {6x0.50}+ {2x(-0.10)}+ {3x2}+ {5x0.65}
= 22.05%

Expected return of Damodar;10+ {6x0.20}+ {2x(-0.90)}+ {3x1.50}+ {5x0.55}
= 16.65%

Actual return = $\lambda_0 + (\lambda_{1.}b_1 + \lambda_{2.}b_2 + \lambda_{3.}b_3 + \lambda_{34}b_4)$

Actual return of Madhav : 18.85+ {(−1)x1.50}+ {1x(−1)}+ {(−1)x0.20}+ {1x0.25} = 16.40%

Alternative way : Actual return of Madhav :
= 10+ {5x1.50}+ {3x(−1)}+ {2x0.20}+ {6x0.25}=16.40%

Actual return of Keshav;25.90+ {(−1)x2}+ {1x(−0.05)}+ {(−1)x0.50}+ {1x0.50} = 23.85%

Alternative way : Actual return of Keshav :
= 10+ {5x2}+ {3x(−0.05)}+ {2x0.50}+ {6x0.50}
= 23.85

Actual return of Krishna; 22.05+ {(−1)x0.50}+ {1x(−0.10)}+ {(−1)x2}+ {1x0.65} = 20.10%

Actual return of Damodar; 16.65+ {(−1)x0.20}+ {1x(−0.90)}+ {(−1)x1.50}+ {1x0.55} = 14.60%

Q. No. 110

Mr. X owns a portfolio with the following characteristics:

	Security A	Security B	Risk Free security
Factor 1 sensitivity	0.80	1.50	0
Factor 2 sensitivity	0.60	1.20	0
Expected Return	15%	20%	10%

It is assumed that security returns are generated by a two factor model.

(i) If Mr. X has Rs. 1,00,000 to invest and sells short Rs. 50,000 of security B and purchases Rs. 1,50,000 of security A what is the sensitivity of Mr. X's portfolio to the two factors?

(ii) If Mr. X borrows Rs. 1,00,000 at the risk free rate and invests the amount he borrows along with the original amount of Rs. 1,00,000 in security A and B in the same proportion as described in part (i), what is the sensitivity of the portfolio to the two factors?

(iii) What is the expected return premium of factor 2? (8 Marks) **(June,2009)**

Answer (i) : W_1 = [1,50,000 / (1,50,000 − 50000)] = 1.50
W_2 = [−50000 / (1,50,000 − 50,000)] = − 0.50

Portfolio sensitivity (Beta) of Factor 1: 1.50 x 0.80+ (−0.50) x 1.50 = 0.45

Portfolio sensitivity (Beta) of Factor 2: 1.50 x 0.60+ (−0.50) x 1.20 = 0.30

(ii) The proportion of investments between A and B [in part (i) of the question] : 1.50 : − 0.50. It means the amount of short sale of B should be 1/3rd of investment in A.

Let Investment in A = x

$x = 2,00,000 + 1/3\ x$

$x = 3,00,000$

It means we have to invest Rs. 3,00,000 in A and go for short sale of Rs.1,00,000 in B.

$W_1 = [3,00,000 / (3,00,000 - 1,00,000 - 1,00,000)] = 3$

(It is the weight of investment in A)

$W_2 = [-1,00,000 / (3,00,000 - 1,00,000 - 1,00,000)] = -1$

(It is the weight of short sale of B)

$W_3 = [-1,00,000 / (3,00,000 - 1,00,000 - 1,00,000)] = -1$

(It is the weight of risk free borrowings)

Portfolio sensitivity (Beta) of Factor 1:

$3 \times 0.80 + (-1) \times 1.50 + (-1) \times 0 = 0.90$

Portfolio sensitivity (Beta) of Factor 2:

$3 \times 0.60 + (-1) \times 1.20 + (-1) \times 0 = 0.60$

(iv) Expected return = Risk Free
+ Expected return premium (also called as risk premium)

Let the value of factor 1 = X

Let the value of factor 2 = Y

Security A: $15 = 10 + 0.80X + 0.60Y$

Security B: $15 = 10 + 1.50X + 1.20Y$

Solving the equations we find that X = 0. It means the premium is coming only from factor 2.

Security A : Total expected return = 15 Risk free = 10
Expected return premium = 5

Security B : Total expected return = 20 Risk free = 10
Expected return premium = 10

SHARPE INDEX MODEL

William Sharpe has developed a simplified variant of the Markowitz[17] model[18]. Sharpe's model is known as Sharpe's Single Index Model. Its data requirement is quite less than that of Markowitz model. Where Markowitz's total requirement of data is [2n + n(n−1)/2], the corresponding requirement of Sharpe's Single Index model is 3n + 2. For example, if portfolio consists of 100 securities, Markowitz requires 5150 inputs, Sharpe requires only 302. The following table illustrates the data inputs for Markowitz model :

No. of securities in the portfolio	No. of security means	No. of security SDs	No. of coefficient of correlations
2	2	2	1
3	3	3	3
4	4	4	6
5	5	5	10
10	10	10	45
20	20	20	190
100	100	100	4950

The following table illustrates the data inputs for Sharpe's Single Index model:

No. of securities in the portfolio	No. of securities means	No. of security SDs	No. of security Betas	Return from market portfolio	Variance of returns from market portfolio	Total
2	2	2	2	1	1	8
3	3	3	3	1	1	11
4	4	4	4	1	1	14
5	5	5	5	1	1	17
10	10	10	10	1	1	32
20	20	20	20	1	1	62
100	100	100	100	1	1	302

Sharpe's Single Index model is explained as follows:

(i) Sharpe divides the security return into two parts (a) the first part, referred as Alpha, it is the return from the security when the return from the market portfolio is zero (b) the second part of the return is influenced by all those factors which affect the return from the market

[17] Dissertation Advisor of Sharpe

[18] Both Markowitz and Sharpe shared Nobel Prize in Economics in 1990.

portfolio i.e. the second part is dependent on the return from the market portfolio.

Security return = security Alpha + (security Beta x RM)

Portfolio return = Portfolio Alpha + (Portfolio Beta x RM)

Where,

Portfolio Beta = Portfolio Beta = $W_1\beta_1 + W_2\beta_2 + W_3\beta_3 + \cdots\cdots. + W_n\beta_n$

Portfolio Alpha = $W_1Alpha_1 + W_2Alpha_2 + W_3Alpha_3 + \cdots\cdots + W_nAlpha_n$

(ii) Portfolio Risk i.e. Portfolio variance =

Market variance x (Portfolio Beta)^2 +
[$(W_1)^2$ x (unsystematic risk of security$_1$)] +
[$(W_2)^2$ x (unsystematic risk of security$_2$)]+
[$(W_3)^2$ x (unsystematic risk of security$_3$)]+ ……………………….+
[$(W_n)^2$ x (unsystematic risk of security$_n$)]

(iii) Covariance$_{1,2}$ = Beta$_1$.Beta$_2$.market variance.

Example : Beta of security A = 1.20
Beta of Security B = 1.10
Market variance = 10.

Calculate covariance between A and B.
Calculate covariance between A and market.
Calculate covariance between B and Market

Answer:

Covariance between A and B = 1.20 x 1.10 x10 = 13.20
Covariance between A and market = 1.20 x 1 x10 = 12.00
Covariance between B and market = 1.10 x 1 x10 = 11.00

(iv) R = Coefficient of determination = r^2

R explains the proportion of systematic risk (out of total risk) when the risk is calculated as variance.

Example : Calculate Beta of A from the following data:

	Security A	Security B	Market
R	0.16	0.17	
Total Variance	7	6	5

Answer

Systematic risk (of A)= 0.16 x 7 = 1.02
1.02 = Beta^2 x 5
Beta = 0.45

(v) The model does not calculate the coefficients of correlation between each pair of the securities as it holds that the coefficient of correlation among various securities get reflected in the market variance

Q. No. 111: A portfolio manager analyzes 70 stocks. She wants to draw efficient frontier under Markowitz's portfolio theory. How many estimates of expected returns, variances and co-variances are to be calculated? How your answer will change if you follow Sharpe's Single Index approach?

Answer : Markowitz

No. of securities in the portfolio	No. of security means	No. of security SDs	No. of co-variances
70	70	70	70(69)/2 = 2415

Sharpe's Index Model :

No. of securities in the portfolio	No. of securities means	No. of security SDs	No. of security Betas	Return from market portfolio	Variance of returns from market portfolio
70	70	70	70	1	1

Q.No.112 : The following are the estimates for two stocks :

Stock	Expected Return	Beta	Residual SD
A	12.50%	0.84	29%
B	18.60%	1.26	44%

Market SD is 23% and the risk free rate of return is 7.60%.
What is the SDs of A and B?
Suppose you construct a portfolio with following proportions :
A 0.28 B 0.46 T bIll 0.26
Compute the expected return, Standard Deviation, Beta and nonsystematic Risk of the port folio.
Answer :
Systematic risk of a security (Based on SD) = Security's Beta x Market SD
Systematic risk of A (Based on SD) = 0.84 x 0.23 = 0.1932
SD of A = 0.1932 + 0.2900 = 0.4832 =48.32%
Systematic risk of B (Based on SD) = 1.26 x 0.23 = 0.2898
SD of B = 0.2898 + 0.4400 = 0.7298 = 72.98%

Expected return of portfolio = 12.50x0.28 + 18.60x0.46 + 7.60x0.26 = 14.032%
Portfolio Beta = 0.84x0.28 + 1.26x0.46+ 0x0.26 = 0.8148
Unsystematic risk of A (based on Variance) = $(0.4832)^2$ – $(0.1932)^2$ = 0.1962
Unsystematic risk of B (based on Variance) = $(0.7298)^2$ – $(0.2898)^2$ = 0.4486

Portfolio Variance =

Market variance x (Portfolio Beta)^2 +
[(W$_1$)^2 x (unsystematic risk of security$_1$ based on variance] +
[(W$_2$)^2 x (unsystematic risk of security$_2$) based on variance]+
[(W$_3$)^2 x (unsystematic risk of security$_2$) based on variance]

$= (0.23)^2 x(0.8148)^2 + (0.28)^2 x(0.1962) + (0.46)^2 x(0.4486) + (0.26)^2 x0$

$= 0.1454$

Portfolio SD = $\sqrt{0.1454} = 0.38$

Unsystematic risk of Portfolio (Based on variance)

$=(0.28)^2 x(0.1962) + (0.46)^2 x(0.4486) + (0.26)^2 x0 = 0.11$

Q.No.113 : The coefficient of correlation between returns of market and those of Security A is 1.00. RM = 10% . Expected return of Security A is 8%. Estimate Beta of Security A.

Hint : Let the market return = X   Let the security A return = Y

X	x	x^2	Y	y	xy
12	2	4	9.60	1.60	3.20
11	1	1	8.80	0.80	0.80
10	0	0	8.00	0	0
9	-1	1	7.20	-0.80	0.80
8	-2	4	6.40	-1.60	3.20
$\sum X = 50$		$\sum x^2 = 10$	$\sum Y = 40$		$\sum xy = 8$

Covariance = $\sum xy / n = 8/5 = 1.60$

Market variance = $\sum x^2 / n = 10/5 = 2$

Security A's Beta = covariance / market variance = 1.60/ 2 = 0.80

Q. No. 114 : Calculate the expected returns, alpha values and total variances for the following stocks.

Asset	Expected return %	Beta	Residual SD%
Stock A	19.50	1.28	59
Stock B	18.40	1.81	70
Stock C	16.90	0.72	63
Stock D	12.10	1.01	58

RM = 16.60%. Market SD(%) = 23.50.   RF rate = 9%

Answer : Expected return = RF + Beta(RM-RF)

Expected return of A = 9 + 1.28(16.60 -9) = 18.73%

Expected return of B = 9 + 1.81(16.60 -9) = 22.76%

Expected return of C = 9 + 0.72(16.60 -9) = 14.47%

Expected return of D = 9 + 1.01(16.60 -9) = 16.68%

Alpha = Security return (Likely) – RMxBeta
Alpha of A = 19.50 – 16.60x1.28 = −1.748
Alpha of B = 18.40 – 16.60x1.81 = − 11.646
Alpha of A = 16.90 – 16.60x0.72 = + 4.948
Alpha of A = 12.10 – 16.60x1.01 = −4.666

Systematic risk of a security (Based on SD) = Security's Beta x Market SD
Systematic risk of A (Based on SD) = 1.28 x 0.2350 = 0.3008
Systematic risk of B (Based on SD) = 1.81 x 0.2350 = 0.4254
Systematic risk of C (Based on SD) = 0.72 x 0.2350 = 0.1692
Systematic risk of D (Based on SD) = 1.01 x 0.2350 = 0.2374
Unsystematic risk of A (Based on SD) = 0.59
Unsystematic risk of B (Based on SD) = 0.70
Unsystematic risk of C (Based on SD) = 0.63
Unsystematic risk of D (Based on SD) = 0.58

	SD	Variance
A	0.3008 + 0.59 = 0.8908	0.7935
B	0.4254 + 0.70 = 1.1254	1.2665
C	0.1692 + 0.63 = 0.7992	0.6387
D	0.2374 + 0.58 = 0.8174	0.6681

Q. No. 115 : The following are the details of three stocks :

Stock	Market capitalization	Beta	Risk Premium	SD %
A	Rs. 4.000 Billion	1.05	11.00%	38
B	Rs 1.356 Billion	0.27	5.00%	36
C	Rs.1.644 Billion	1.79	17.50%	57

Market SD = 24%
What is the mean (%) of risk premium of the three stocks?
What is the covariance between stock A and the market?
Decompose the variance of C into systematic and unsystematic components

Answer
W_1 = 4/(4+ 1.356+ 1.644) = 0.5714
W_2 = 1.356/(4+ 1.356+ 1.644) = 0.1937
W_1 = 1.644/(4+ 1.356+ 1.644) = 0.2349

Mean (%) of risk premium of 3 stocks =
11.00x0.5714 + 5x0.1937 + 17.50x0.2349 = 11.36
Beta = Covariance / Market variance
Beta A = 1.05 =Covariance / $(0.24)^2$
Covariance of A = 0.06048
Variance of C = 0.3249
Systematic risk of C (Based on Variance) = $(1.79)^2$ x $(0.24)^2$ = 0.1846
Unsystematic risk of C (Based on Variance) = 0.3249 – 0.1846 = 0.1403

Q. No. 116: Following are the details of a portfolio consisting of three shares :

Share	Portfolio weight	Beta	Expected return	Total variance
A	0.20	0.40	14	0.015
B	0.50	0.50	15	0.025
C	0.30	1.10	21	0.100

SD of market portfolio returns = 10%

(a) Find the portfolio Beta.
(b) Residual variance of each of three shares
(c) Portfolio Variance using Sharpe Index Model
(d) Expected return of the portfolio
(e) Portfolio variance (on the basis of Modern portfolio theory given by Markowitz). You are given the following additional data.

Covariance (A,B) = 0.030 Covariance (A,C) = 0.020
Covariance (B,C) = 0.040

Answer

(a) Portfolio Beta = 0.40x0.20 + 0.50x0.50 + 1.10x0.30 =0.66

(b)

	Total variance	Systematic risk (variance)	Residual variance
A	0.015	$(0.40)^2$ x0.01 =0.0016	0.0134
B	0.025	$(0.50)^2$ x0.01 =0.0025	0.0225
C	0.100	$(1.10)^2$ x0.01 =0.0121	0.0879

(c) Portfolio variance (Sharpe) :

Market variance x $(\text{Portfolio Beta})^2$ +
$[(W_1)^2$ x (unsystematic risk of security_1 based on variance] +
$[(W_2)^2$ x (unsystematic risk of security_2) based on variance]+
$[(W_3)^2$ x (unsystematic risk of security_2) based on variance]

$= (0.10)^2 x(0.66)^2 + (0.20)^2 x(0.0134) + (0.50)^2 x(0.0225)+ (0.30)^2x(0.0879)$
= 0.01842

(d) Expected return on portfolio: 14x0.20 + 15x0.50 + 21x0.30 = 16.60%

(e) Portfolio variance (Markowitz):

$= (0.20)^2x(0.015) + (0.50)^2x(0.025) + (0.30)^2x(0.100) +$
$2(0.20)(0.50)(0.030) + 2(0.20)(0.30)(0.020) + 2(0.50)(0.30)(0.040)$
= 0.03625

Q. No. 117 : Given that risk premium on market portfolio is 10%, RF is 6% and market SD is 0.30, complete the following table :

Share	Expected return	Standard deviation	Beta	Residual variance
A	15%			0.0375
B	14%	0.40		

Answer :

A : Expected return = 15 = RF + Beta (RM-RF)

15 = 6 + Beta(10) Beta = 0.90

Systematic Risk (based on variance) : Market variance x $(Beta)^2$

$= (0.30)^2 \times (0.90)^2 = 0.0729$

Unsystematic Risk (based on variance) : 0.0375

Total variance = 0.0729 + 0.0375 = 0.1104

SD = $\sqrt{0.1104}$ = 0.3323

B : Expected return = 14 = RF + Beta (RM-RF)

14 = 6 + Beta(10) Beta = 0.80

Systematic Risk (based on variance) : Market variance x $(Beta)^2$

$= (0.30)^2 \times (0.80)^2 = 0.0576$

Unsystematic Risk (based on variance) = $(0.40)^2 - 0.0576 = 0.1024$

Q. No. 118 : Assuming RM as 15% and market variance 320, find the expected return and variance of a portfolio the details of which are given below:

Security	Weight	Alpha	Beta
A	0.25	2.10	1.65
B	0.15	3.60	0.55
C	0.35	1.55	0.75
D	0.25	0.70	1.40

Residual variance of A,B,C and D is 380, 140, 310 and 385 respectively.

Answer :

Portfolio Alpha : 2.10x0.25 + 3.60x0.15 + 1.55x0.35 + 0.70x0.25 = 1.7825

Portfolio Beta : 1.65x0.25 + 0.55x0.15 + 0.75x0.35 + 1.40x0.25 = 1.1075

Portfolio Return = Portfolio Alpha + RM x Portfolio Beta

= 1.7825 + 15x1.1075 = 18.395

Portfolio Variance : $(320) \times (1.1075)^2 + (0.25)^2 \times (380) + (0.15)^2 \times (140)$

$+ (0.35)^2 \times (310) + (0.25)^2 \times (385) = 481.4375$

Q No. 119 : An investor wants to build a portfolio with the following four stocks. With the given details, find out his portfolio return and portfolio variance. The investment is equally spread over the four stocks. Market return = 10. Market variance = 27

Security	Residual Variance	Alpha	Beta
A	45.00	0.50	0.90
B	130.00	2.50	1.30
C	199.00	1.50	1.40
D	53.00	2.50	2.10

Answer

Portfolio Alpha : 0.50x0.25 + 2.50x0.25 + 1.50x0.25 + 2.50x0.25 = 1.75

Portfolio Beta : 0.90x0.25 + 1.30x0.25 + 1.40x0.25 + 2.10x0.25 = 1.425

Portfolio Return = Portfolio Alpha + RM x Portfolio Beta

= 1.75 + 10x1.425 = 16%

Portfolio Variance :

$= (27)(1.425)^2 + (0.25)^2 x(45) + (0.25)^2 x(130) + (0.25)^2 x(199) + (0.25)^2 x(53)$

= 81.5175

Q. No. 120: An investor wants to build a portfolio with the following four stocks. With the given details, find out his portfolio return and portfolio variance. The investment is equally spread over the four stocks. Market return = 10. Market variance = 5

Security	Residual Variance	Alpha	Systematic variance
A	3.00	- 0.08	5.10
B	5.50	0.11	2.20
C	1.10	0.02	3.30
D	2.30	-0.15	3.20

Answer

Systematic variance = $[(Beta)^2 x \text{ Market Variance}]$

A : 5.10 = $[(Beta)^2 x\ 5]$ Beta = 1.01

B : 2.20 = $[(Beta)^2 x\ 5]$ Beta = 0.66

C : 3.30 = $[(Beta)^2 x\ 5]$ Beta = 0.81

D : 3.20 = $[(Beta)^2 x\ 5]$ Beta = 0.80

Portfolio Alpha : 0.25(-0.08 + 0.11 + 0.02 -0.15) = -0.025

Portfolio Beta : 0.25(1.01 + 0.66 + 0.81 + 0.80) = 0.842

Portfolio return = Portfolio Alpha + RM x Portfolio Beta

= -0.025 + 0.842x10 = 8.395

Portfolio Variance : $5x(0.842)^2 + (0.25)^2 x(3.00+ 5.50+ 1.10+ 2.30)$

=4.28857

Q. No. 121: 1,000 Equity shares are listed on a stock exchange. Each share has a Beta of 1 and SD of 0.40. A portfolio manager selected 40 shares after his research; 20 shares had Alpha of 3% and another 20 had Alpha of −3%. He borrowed the negative Alpha shares of Rs.1,00,000 each, sold these shares and invested the amount equally in 20 shares having positive Alpha. Calculate the portfolio return and variance.

Answer : Portfolio return : Rs.1,20,000

$$\text{Variance} = [(0.40)^2 .(1{,}00{,}000)^2] \times 20 + [(0.40)^2 .(-1{,}00{,}000)^2] \times 20$$

$$= 64000 \text{ Million}$$

> **Teaching note** : It is a zero investment portfolio. In this case, the weights are "the amounts of investments" and not the ratio of investment.

SHARPE'S OPTIMAL PORTFOLIO

A Portfolio is defined as optimal portfolio if its Sharpe Ratio is highest. Sharpe ratio is defined as risk premium per unit of total risk. Putting differently:

Sharpe ratio = (Expected return − Risk Free return)/ SD

> The difference between expected return and risk free return is referred as risk premium. It is also refereed as "Mean excess return".
>
> In the following paragraphs, we shall be referring the Sharpe ratio as "Risk premium/SD".
>
> By expected return we mean, the return that the investor expects on the basis of historical data.

The following steps are required for portfolio construction under Sharpe's approach:

(i) Calculate the ratio of "Risk Premium/Beta" for each security.

(ii) Rank the ratios calculated under (i) in descending order.

(iii) Calculate the ratios of Beta^2 / Residual variance for each security.

(iv) Multiply (ii) and (iii) [{(Risk premium/Beta) x(Beta^2 / Residual variance)} = {(Risk premium x Beta)/(Residual variance)]

(v) Find cumulative value of (iii)

(vi) Find cumulative values of (iv)

(vii) Calculate Ci for each security.

$C_1 = \dfrac{\text{Market variance x (Risk premium x Beta)/(Residual variance)}}{1 + \text{Market variance (Beta}^2\text{ / Residual variance)}}$

$C_2 = \dfrac{[\{\text{(Market variance)x(Risk premium x Beta)}\}\text{/(Residual variance)}]}{1 + [\text{Market variance (Beta}^2\text{ / Residual variance)}]}$

And so on.

The value of (i) (Risk premium x Beta)/(Residual variance) and (ii) (Beta^2 / Residual variance) are to be taken from cumulative columns.

The highest Ci value is referred as C*. It is the cut-off rate. Security with C* value and the securities before this security are to be included in the portfolio, others are rejected.

vii)

The next step is to calculate the weights. For this purpose, we have to calculate Zi.

Zi=[Beta/residual variance] X [(Risk premium/Beta) - C*]
W_1 = Zi /(Zi + Zii + Ziii + ……….. + Z_n)

W_2 = Zii /(Zi + Zii + Ziii + ……….. + Z_n)

And so on.

[All these steps are simplified version of advanced mathematical derivations. Such derivations are beyond the scope of this note]

Reference :
Study Material on SFM page 7.28 -7.30 (Board of Studies, ICAI)

Q . No. 122 Mr. Ramesh wants to invest in stock market. He has got the following information about individual securities:

Security	Expected return	Beta	SD^2_{ci}
A	15	1.5	40
B	12	2	20
C	10	2.5	30
D	09	1	10
E	08	1.2	20
F	14	1.5	30

Market Index variance is 10 and the risk free rate of return is 7%. What should be the optimum portfolio assuming no short sales.
(May, 2010) (10 Marks)

Teaching note – **not required in the exam**: SD^2 c_i given in the last question of the question refers to Residual variance. It is evident from the following observations:

(i) Systematic risk of B = 2^2 x 10 = 40. The figure given in the last column is 20. The figure given in the last column cannot be total risk as total risk cannot be less than systematic risk. It means the figures given in the last column represent residual risk.

(ii) Systematic risk of C = 2.5^2 x 10 = 62.50. The figure given in the last column is 30. The figure given in the last column cannot be total risk as total risk cannot be less than systematic risk. It means the figures given in the last column represent residual risk.

Answer :
Note: It is assumed that the ci_{given} in the question is ei.
(ei refers to residual variance)

Security	Risk premium	Beta	Risk Premium/Beta
A	8	1.5	5.33
B	5	2	2.50
C	3	2.5	1.20
D	2	1	2
E	1	1.2	0.83
F	7	1.5	4.67

Secu-rity	(Risk Premium)/ (Beta)	($Beta^2$) / (Residual Variance)	(Risk premium x Beta) / (Residual variance)	Cum. Value of ($Beta^2$ / Residual Variance)	Cum. Value of (Risk premium x Beta)/ (Residual variance)	C
A	5.33	0.05625	0.30	0.05625	0.30	1.92
F	4.67	0.07500	0.35	0.13125	0.65	2.8110
B	2.50	0.20000	0.50	0.33125	1.15	2.667
D	2.00	0.10000	0.20	0.43125	1.35	2.5414
C	1.20	0.20833	0.25	0.63958	1.60	2.1635
E	0.83	0.07200	0.06	0.71158	1.66	2.0452

Zi=[Beta/residual variance] X [(Risk premium/Beta) – C*]

= [1.5/40] X [(5.33-2.8110)] = 0.09446625
Zii = [1.5/30] x [4.67 -2.8110] = 0.09295
0.280335

W_1 = 0.0944625/(0.0944625 + 0.09295) = 0.50
W_2 = 0.09295 /(0.0944625 + 0.09295) = 0.50
Mr. Ramesh should invest 50% of his funds in A and 50% in F.

Teaching note – not to be given in the exam :
Security A :
variance = $(1.50)^2.(10)$ + 40 = 62.50
SD = 7.91
Sharpe ratio : 8/7.91 = 1.0113

Portfolio :
portfolio expected return = = 14.50
Portfolio Beta = 1.5
Portfolio systematic risk (Variance) = 1.5 x 1.5 x 10 = 22.50
Portfolio Unsystematic risk (variance) =
$(0.5)^2.(40) + (0.5)^2.(30)$ = 17.50
Portfolio total risk (variance) = 40
Portfolio SD = 6.3246
Sharpe ratio = 7.5/6.3246 1.19
A has got highest Sharpe ratio among all the six securities. Portfolio's Sharpe ratio is more than even A's Sharpe ratio.

Q. No. 123:

	Security A	Security B	Market
Alpha	3	-1	
Beta	1.10	1.20	?
Return	?	?	10%
R	.40	?	
Residual SD	9%	12%	?
Variance	?	?	?

Answer

- Beta of Market = 1
- Residual variance of market = 0
- Required return of A = 3 + (1.1 x 10) = 14 %
- Required return of B = -1 + (1.2 x 10) = 11 %

- Systematic Risk of A = $(1.10)^2$.(market variance)

=1.21(Market Variance)

0.40[1.21(Market variance) + 0.81] =1.21(Market-Variance)

- 0.484(Market Variance) + 0.324 = 1.21(Market-Variance)
- -0.726(Market Variance) = - 0.324
- Market variance = 0.45%
- Systematic Risk of A = $(1.10)^2$.(0.45) = 0.5445 %
- Unsystematic Risk of A = 0.81%
- Total risk of A =1.3545%

- Systematic Risk of B = $(1.20)^2$.0.45 = 0.648%
- Unsystematic risk of B : 1.44%
- Total risk of B = 2.088
- R of B = 0.648/(0.648 + 1.44) = 0.31

	Security A	Security B	Market
Alpha			
Beta			1
Return	14	11	
R		0.31	
Residual SD	9%	12%	0
Variance	1,3545%	2.088	0.45%

EXTRA PRACTICE (MUST DO)

Q. No.124: The average return of a portfolio is 15% p.a. and SD is 7.50. Find the probability of at least no loss.

Answer

Z = (0-15) / 7.50 = -2   Probability = 0.4772 + 0.05 = 0.9772

Q. No. 125 : Parthasarthy Ltd is a company that has diversified into five different industries in five different countries. The investments are each approximately equal in value. The company's objective is to reduce risk through diversification, and it believes that the return on any investment is not correlated with the return on any other investment. The estimated risk and return (in present value terms) of the five investments are shown below:

Investment	Risk (% standard deviation)	Return
1	8	14
2	10	16
3	7	12
4	4	9
5	16	22

Estimate the risk and return of the portfolio of five investments, and briefly explain the significance of your results.

Answer

Portfolio Return

=(0.20x0.14) + (0.20x0.16) + (0.20x0.12) + (0.20x0.09) + (0.20x0.22) =0.146 = 14.60%

Portfolio Variance = $(0.20)^2 x (0.08)^2 + (0.20)^2 x (0.10)^2 + (0.20)^2 x (0.07)^2 + (0.20)^2 x (0.04)^2 + (0.20)^2 x (0.16)^2 = 0.00194$

Portfolio SD = 0.044 = 4.44%

Weighted average SD = (0.20)(0.08) + (0.20)(0.10) + (0.20)(0.07) + (0.20)(0.04) + (0.20)(0.16) = 0.09 = 9%

$$\text{Portfolio Gain (\%)} = \frac{\text{Weighted average SD} - \text{Portfolio SD}}{\text{Weighted average SD}} \times 100$$

$$= \frac{9 - 4.44}{9} \times 100 = 50.67$$

The portfolio has reduced the risk significantly.

Q. No. 126: An investor is holding 1000 shares of Fatlass company. Presently the rate of dividend being paid by the company is Rs. 2 per share and the share is being sold at Rs.25 per share. However, several factors are likely to change during the course of the year as indicated below :

	Existing	Revised
Risk free rate	12%	10%
Market risk premium	6%	4%
Beta	1.4	1.25
Expected growth rate	5%	9%

In view of the above factors whether the investor should buy, hold or sell the shares? Why? **(May, 2003)**

Answer

Existing factors : Ke = 12 + 1.40(6) =20.40

$$\text{Value of the share} = \frac{2(1.05)}{0.2040 - 0.05} = 13.64$$

Revised factors : Ke = 10 + (1.25)(4) = 15

$$\text{Value of share} = \frac{2(1.09)}{0.15 - 0,09} = 36.33$$

If the revised estimates are reliable, the share may be purchased as its worth (Rs. 36.33) is more than current price of Rs.25. Its market price may go up in near future.

If the existing factors are likely to continue, the share may be sold as its worth is less than current market price. Hence, the market price may go down in near future.

EXTRA PRACTICE QUESTIONS (OPTIONAL)

Q. No. 127: RF = 10%, RM = 20%, Beta of a security = 1.50. Payout ratio = 60%. Earning per share in the year just ended Rs.10. The company earns a rate of return of 20% on its retained earnings. Find the equilibrium value of the share. If it is available in the market at Rs. 20 and it is expected that after 1 year it will be quoted in the market at 25% above the present equilibrium value, what is return for holding the share for 1 year?

Answer

Ke = RF + Beta(RM-RF)

= 10 + 1.50(20-10) = 25%

g = b.r, Where,

g = rate of growth

b = ratio of retained EPS to EPS

r = rate of return on retained earnings

$$\text{Equilibrium price of share} = P = \frac{D_1}{Ke - g} = \frac{6(1.08)}{0.25 - 0.08} = 38.18$$

Equilibrium price of the share after 1 year : 38.18(1.25) = 47.65

Return for holding the share for 1 year = [47.65/20) – 1] = 138.24%

Q. No.128: Mr Brijwasi invested Rs.1,00,000 in the equity shares of three companies – Girdhari Ltd., Banwari Ltd. and Murari Ltd. in the ratio of 4:3:3. Given the following Variance-covariance Matrix for the three companies, find the portfolio variance :

	Equity shares of Girdhari Ltd.	Equity shares of Banwari Ltd.	Equity shares of Murari Ltd.
Equity shares of Girdhari Ltd.	46	-22	12
Equity shares of Banwari Ltd.	-22	32	21
Equity shares of Murari Ltd.	12	21	18

Answer

$$\begin{aligned}\text{Portfolio variance} = {} & (0.40)^2(46) + (0.30)^2(32) + (0.30)^2(18) + \\ & 2(0.40)(0.30)(-22) + \\ & 2(0.40)(0.30)(12) + \\ & 2(0.30)(0.30)(21) = 13.24\end{aligned}$$

Q. No. 129: Hari has his investments in equity shares of four companies :

	No. of shares held	Face value	Beta	Market price	Exp. Return
Govind Ltd	10,000	Rs. 5	1.2	Rs. 30	19 %
Mansukha Ltd	20,000	Rs.10	0.80	Rs.50	13 %
Parthsarthy Ltd	30,000	Rs. 2	0.90	Rs.40	14%
Krishna Ltd	40,000	Re. 1	1.50	Rs.10	20%

RF = 5 %   RM = 15 %. Find the risk of the portfolio relative to the market risk. Should the composition of the portfolio changed?

Answer

	Investment	Proportion
Govind	10000x30 = 3,00,000	3,00,000 / 29,00,000 = 0.1034
Manshukha	20000x50 = 10,00,000	10,00,000 / 29,00,000 = 0.3449
Parthsarthy	30000x40 = 12,00,000	12,00,000 / 29,00,000 =0.4138
Krishna	40000x10 = 4,00,000	4,00,000 / 29,00,000 = 0.1379
Total	29,00,000	1.00

Risk of portfolio relative to market (Beta) =
1.20(0.1034) + 0.80(0.3449) + 0.90(0.4138) + 1.50 (0.1379)= 0.9793
Required return:

Govind	5 + 1.20(15 – 5)	17 %
Manshukha	5 + 0.80 (15–5)	13 %
Parthsarthy	5 + 0.90 (15 –5)	14 %
Krishna	5 + 1.50 (15 – 5)	20 %

- Mansukha, Parthasarthy and Krishna are neutral securities i.e. the required return from these securities is equal to Expected (likely) return.
- If the investor cannot afford increased risk, there should be no change in the portfolio; if the investor is willing to bear increased risk, he should invest all the funds in Govind.

Q. No. 130: Madhav purchased a share for Rs.100. After 1 year, its value may be Rs.500 (Probability 0.50) or Rs.100 (Probability 0.30) or 0. Find the expected return and SD of the investment.
Answer

Wealth ratio	Return (X)	Probability	pX	x	px^2
500/100 =5	400%	0.50	200	220	24200
100/100 =1	0	0.30	0	-180	9720
0/100 = 0	-100%	0.20	-20	-280	15680
			180		49600

Expected return = 180 SD = $\sqrt{49600}$ = 222.71

Q. No. 131: A portfolio is made of
(i) Government of India securities and,
(ii) Market portfolio

The return of the portfolio is 28% and standard deviation is 5. RF = 6% and RM = 50%.

Coefficient of correlation of return from equity shares Keshav with market returns is 0.50. SD of Keshav's return is 2. Find the expected return of equity shares of Keshav Ltd.

Answer
Assumption : $W_1 = W_2$
Portfolio Return = 0.28 = $W_1(6) + (1-W_1)(50)$
$W_1 = 0.50$ $W_2 = 0.50$

Portfolio SD =

$$\sqrt{(0.50)^2(0)^2 + (0.50)^2(SDm)^2 + 2(0.50)(0.50)(0)(0)(SDm)} = 5$$

Market SD = 10 Market variance = 100

$$r = \frac{\text{Covariance (Market and Keshav)}}{SDm \cdot SDk}$$

$$0.50 = \frac{\text{Covariance (M\&K)}}{10x2} \qquad \text{Covariance (M\&K)} = 10$$

Beta Of Keshav = Covariance / Market variance = 10/100 = 0.10
Expected return of K = RF + Beta (RM-RF) = 6+ 0.10(50 – 6) =10.40%

Q.No.132 An investor in interested in investing the shares of Kunj Bihari Ltd and Ras Bihari Ltd. He borrows the funds for this purpose. What should be risk free rate of his borrowings? Assume that the coefficient of correlation between returns from the two securities is -1. Given :

	Expected return	SD
KB	8%	4%
RB	12%	8%

Answer

If we invest in the ratio of 8:4, i.e. 2/3 : 1/3 in KB and RB respectively, we shall have to bear the minimum risk.

Variance or risk of this portfolio =

$$(2/3)^2(0.04)^2 + (1/3)^2(0.08)^2 + 2(1/3)(2/3)(-1)(0.04)(0.08)$$
$$= 0$$

Return from this portfolio = 8(2/3) + 12(1/3) = 9.33

Risk free rate of return= 9.33%

Q. No. 133: The current market price of share is Rs.50. The expected rate of return is 10%. Expected dividend yield is 6%. What shall be the cum-dividend market price of the share at the year end? Ex-dividend?

Answer

Cum-dividend market price at year end = 50(1.10) = 55

Ex-dividend price = 52

Q. No. 134: The current market price of share is Rs.50. The expected rate of return is 10%. Assuming payout ratio to be zero, what should be the market price of the share after 4 years from today?

Answer

Market price after 4 years from today ; $50(1.10)^4 = 73.21$

Q. No. 135: The current market price of share is 63. The expected rate of return is 10%. Last year the dividend per share was Rs.3. Dividend has been growing at the rate of 5 per annum and this growth is likely to be maintained. At what price you shall be able to buy the share after 3 years from today?

Answer

$$\text{Market price after 3 years} = \frac{3(1.05)^4}{0.10 - 0.05} = 72.93$$

Q. No. 136: An investor selects two equity shares for investment – A and B. Current market price of A is Rs.400. The probability distribution of A's expected price next year is as follows:

Price	Probability
360	0.10
440	0.80
480	0.10

Other data:

	A	B
Correlation with market	0.80	0.20
Expected return	Not known	9%
SD	Not known	12

Market SD=8. Coefficient of correlation between returns from A&B = 0.63

The investor invests 25% in A and 75% in B. What is his expected return? Calculate the Variance and Beta of his portfolio. How your answer will change if he invests 25% in A, 60% in B and 15% in C? Assume C is a risk-free security with expected return of 5%.

Answer

Calculation of expected return and SD of A :

Year end Price	Return (X)	pX	x	px^2
360	-10%	-1	-19	36.10
440	+ 10%	8	+ 1	0.80
480	+ 20%	2	+ 11	12.1
Total		9		49

Expected return of A = 9 SD of returns from A = 7

$$r = \frac{\text{Covariance (Market and A)}}{\text{SDm . SDa}}$$

$$0.80 = \frac{\text{Covariance (M\&A)}}{8 \times 7} \qquad \text{Covariance (M\&A)} = 44.80$$

Beta of A = Covariance (M&A)/ market variance

= 44.80 /64 = 0.70

$$r = \frac{\text{Covariance (Market and B)}}{\text{SDm . SDb}}$$

$$0.20 = \frac{\text{Covariance (M\&B)}}{8 \times 12} \qquad \text{Covariance (M\&B)} = 19.20$$

Beta of B = Covariance (M&B)/ market variance

= 19.20 /64 = 0.30

<u>25% in A and 75% in B</u>

(a) Expected return = 9%

(b) Portfolio Beta = (0.25)(0.70) + (0.75)(0.30) = 0.40

(c) Portfolio Variance=

$(0.25)^2(7)^2 + (0.75)^2(12)^2 + 2(0.25)(0.75)(0.63)(7)(12) = 103.91$

<u>25% in A, 60% in B and 15% in C</u>

a) Expected return = (0,25)(9) + (0.60)(12) + (0.15)(5) = 10.20

(b) Portfolio Beta = (0.25)(0.70) + (0.60)(0.30) + (0.15)(0)= 0.355

(c) Portfolio Variance=

$(0.25)^2(7)^2 + (0.60)^2(12)^2 + (0.15)^2(0)^2 + 2(0.25)(0.60)(0.63)(7)(12) = 70.78$

Q.No.137 : Tex Ltd. never pays dividend. Its equity share is currently selling at $25. Using the following data, find the expected return and standard deviation of expected returns of equity share of Tex Ltd.

Price of equity share after 1 year	Probability of the price
$20	.10
$30	.20
$40	.40
$50	.20
$60	.10

Answer

Price after 1 year	Wealth ratio	Return
20	20/25 = 0.80	−0.20
30	30/25 = 1.20	0.20
40	40/25 = 1.60	0.60
50	50/25 = 2.00	1.00
60	60/25 = 2.40	1.40

X	P	pX	x	x^2	px^2
−0.20	0.10	−0.02	−0.80	0.64	0.064
0.20	0.20	0.04	−0.40	0.16	0.032
0.60	0.40	0.24	0	0	0.000
1.00	0.20	0.20	0.40	0.16	0.032
1.40	0.10	0.14	0.80	0.64	0.064
Total	1.00	0.60			0.192

Expected return = $\sum pX / p = 0.60$
S.D. = 0.44

Q. No. 138: Madhav purchased a share for Rs.100. After 1 year, its value may be Rs.500 (Probability 0.50) or Rs.100 (Probability 0.30) or 0. Find the expected return and SD of the investment.

Answer :

Wealth ratio	Return (X)	Probability	pX	x	px^2
500/100 =5	400%	0.50	200	220	24200
100/100 =1	0	0.30	0	-180	9720
0/100 = 0	-100%	0.20	-20	-280	15680
			180		49600

Expected return = 180 SD = $\sqrt{49600}$ = 222.71

Q. No. 139: Calculate the total variances for the following stocks.

Asset	Beta	Residual SD%
Stock A	1.3	50
Stock B	1.8	64
Stock C	0.7	60
Stock D	1.0	55

Market SD(%) = 20.

Answer

Systematic risk of A: Beta of A x Market Standard Deviation
= 1.30 x 20% = 26%
Total Risk of A (SD of A) = 26 + 50 = 76%

Systematic risk of B: Beta of B x Market Standard Deviation
= 1.80 x 20% = 36.00%
Total Risk of B (SD of B) = 36+ 64 = 100%

Systematic risk of C: Beta of C x Market Standard Deviation
= 0.70 x 20% = 14%
Total Risk of C (SD of C) = 14 + 60 = 74%

Systematic risk of D: Beta of D x Market Standard Deviation
= 1.00 x 20% = 20.00%
Total Risk of D (SD of D) = 20+ 55 = 75%

Q. No. 140: Risk free rate of return is 8 %. The return from market portfolio is expected to be 16 %. Inder Ltd. has just paid a dividend of Rs. 3 per share. The expected growth rate of dividend is 10% p.a. Its equity Beta is 1.50. Using CAPM, find the required return of equity shareholders. What is the present market price per equity share (using the required rate of return calculated by you)?

Answer : Ke = RF + Beta(RM-RF)
= 8 + 1.5(16-8) = 20 %

$$P = \frac{D_1}{Ke - g} = \frac{3(1.10)}{0.20 - 0.10} = Rs.33$$

Q. No. 141: Coefficient of correlation between returns from market portfolio and those from security Zeta is 0.65. SD of Zeta is 0.25 and that of market is 0.20. Find Beta of Zeta.

Answer :

$$r = \frac{Covariance}{(SDx)(SDy)}$$

$$0.65 = \frac{Covariance}{0.25 \times 0.20}$$

Covariance = 0.0325

Beta = (covariance / market variance) = (0.0325) / $(0.20)^2$ = 0.8125

Q. No. 142: Find the expected return of security X on the basis of following data: Risk free rate of return 8%, expected return from market portfolio 12%, SD of X 0.50, SD of market 0.40, relevant correlation 0.80.

Answer

$$r = \frac{Covariance}{(SDx)(SDy)}$$

$$0.80 = \frac{Covariance}{0.50 \times 0.40}$$

Covariance = 0.16

Beta = covariance / market variance = (.16) / $(.40)^2$ =1
Expected return = 8% + 1(12-8) = 12%

Q. No. 143: Given below is information of market rates of Returns and Data from two companies

	Year 2002	Year 2003	Year 2004
Market (%)	12	11	9
Company A (%)	13	11.5	9.8
Company B (%)	11	10.5	9.5

Determine the Beta coefficients of shares of A and B. **(Nov. 2004)**

Answer

Calculation of Beta of A

Let the return from market = X   Let the return from the A Ltd = Y

X	x	Y	y	xy	x^2
12	1.33	13.00	1.5667	2.0837	1.7689
11	0.33	11.50	0.0667	0.0220	0.1089
9	-1.67	9.80	-1.6333	2.7276	2,7889
$\sum X = 32$		$\sum Y$=34.30		4.8333	$\sum x^2$=4.6667

Mean of X = 10.67   Mean of Y = 11.4333

Covariance = $\sum xy/n$ = 4.8333/3 = 1.6111

Market variance = $\sum x^2/n$ = 4.6667/3 = 1.5556

Beta = covariance/Market variance = 1.6111/1.5556 = 1.036

Calculation of Beta of B

Let the return from market = X   Let the return from the B Ltd = Y

X	x	Y	y	xy	x^2
12	1.33	11.00	0.6667	0.8867	1.7689
11	0.33	10.50	0.1667	0.0550	0.1089
9	-1.67	9.50	-0.8333	1.3916	2,7889
$\sum X = 32$		31		2.3333	$\sum x^2$=4.6667

Mean of X = 10.67   Mean of Y = 10.3333

Covariance = $\sum xy/n$ = 2.3333/3 = 0.7777

Market variance = $\sum x^2/n$ = 4.6667/3 = 1.5556

Beta = covariance/Market variance = 0.7777/1.5556 = 0.50

Q. No. 144: A company pays a dividend of Rs.2 per share with a growth rate of 7%. The risk free rate is 9% and the market rate of return is 13%. The company has a beta factor of 1.50. However, due to a decision of the Finance manager, beta is likely to increase to 1.75. Find out the present as well as the likely value of the share after the decision.**(May, 2005)**

Answer

Interpretation : Dividend of Rs.2 per share has just been paid.

Before the decision of the finance manager:

Ke = RF + Beta(RM-RF) = 9 + 1.50(13-9) =15%

$$P = \frac{D_1}{Ke - g} = \frac{2.14}{0.15 - 0.07} = 26.75$$

After the decision of the finance manager :
Ke = RF + Beta (RM-RF) = 9 + 1.75(13-9) = 16%

$$P = \frac{D_1}{Ke - g} = \frac{2.14}{0.16 - 0.07} = 23.78$$

Alternative solution ;
Interpretation : Dividend of Rs.2 per share is to be paid.
Before the decision of the finance manager:

Ke = RF + Beta(RM-RF) = 9 + 1.50(13-9) =15%

$$P = \frac{D_1}{Ke - g} = \frac{2.00}{0.15 - 0.07} = 25.00$$

After the decision of the finance manager :
Ke = RF + Beta (RM-RF) = 9 + 1.75(13-9) = 16%

$$P = \frac{D_1}{Ke - g} = \frac{2.00}{0.16 - 0.07} = 22.22$$

Teaching note- not to be given in the exam. The first answer is preferred because the question says "company pays" (present tense), it can be interpreted as has been paid (present tense). In the second answer, we are interpreting as the company is going to pay (future tense), while the question is talking about present tense.

Q. No. 145: RF is 7%, RM is15%, market SD 0.20, SD of security K = 0.24, r between returns from security K and those from the market is 0.68. Find the required return from security K.

Answer

$$r = \frac{\text{Covariance}}{(SDx)(SDy)}$$

$$0.68 = \frac{\text{Covariance}}{0.20x0.24}$$

Covariance = 0.03264

Beta = (covariance / market variance) = 0.03264 / $(0.20)^2$ = 0.816

Ke = RF + Beta(RM-RF) = 7 + 0.816(15-7) = 13.53

Q. No. 146 : The Beta coefficient of Target Ltd is 1.4. The company has been maintaining 8% rate of growth of dividends and earnings . The last year dividend was Rs.4.00 per share . Return on government securities is 10%. Return on market portfolio is 15%. The current market price is Rs. 36. What will be the equilibrium price per share of target Ltd.? Would you advise purchasing the share? **(May, 1997)**

Answer :

Ke = RF + Beta (RM-RF) = 10 + 1.40(15-10) = 17%

$$P = \frac{D_1}{Ke - g} = \frac{4.32}{0.17 - 0.08} = 48$$

The worth of the share is Rs.48 while it is available in the market at Rs.36.
Purchase of the share is recommended.

Q. No. 147: An investor is seeking the price to pay for a security, whose SD is 3%, r for the security with market is 0.80. Market SD = 2.2%. RF = 5.20% . RM =9.80% Find the required return. **(May,1998)**

Answer :

$$r = \frac{\text{Covariance}}{(SDx)(SDy)}$$

$$0.80 = \frac{\text{Covariance}}{0.03 \times 0.022}$$

Covariance = 0.000528

Beta = (covariance / market variance) = 0.000528 / $(0.022)^2$ = 1.0909
Ke = RF + Beta(RM-RF) = 5.20 + 1.0909(9.80-5.20) = 10.22%

Q No. 148: The distribution of return of security F and the market portfolio P is given below :

Probability	Return % F	Return % P
0.30	30	-10
0.4	20	20
0.30	0	30

You are required to calculate the expected return of security F and the market portfolio P, the covariance between the market portfolio and security and Beta of the security. **(May 2006)**

Answer

Calculation of Beta of F

Let the return from market = X Let the return from the F Ltd = Y

X	p	pX	x	px^2	Y	p	pY	y	pxy
-10	.30	-3	-24	172.80	30	.30	9	13	-93.60
20	.40	8	6	14.40	20	.40	8	3	7.20
30	.30	9	16	76.80	0	.30	0	-17	-81.60
		∑pX= 14		$\sum px^2$ =264			∑pY =17		∑pxy = -168

Expected return of the market portfolio = 14%

Expected return of the security = 17%

Covariance = -168/1 = -168

Market variance = 264

Beta = -168/264 = -0.636

Q. No. 149: The historical rates of return (%) of two securities are given below. Calculate covariance and coefficient of correlation of two securities :

Year	1	2	3	4	5	6	7	8	9	10
Security A	12	8	7	14	16	15	18	20	16	22
Security B	20	22	24	18	15	20	24	25	22	20

(May 2007)

Answer

Let the return from A = X Let the return from B = Y

Calculation of coefficient of correlations

X	x	x^2	Y	y	y^2	xy
12	-2.80	7.84	20	-1.00	1	2.80
8	-6.80	46.24	22	1.00	1	-6.8
7	-7.80	60.84	24	3.00	9	-23.4
14	-0.80	0.64	18	-3.00	9	2.4
16	1.20	1.44	15	-6.00	36	-7.2
15	0.20	0.04	20	-1.00	1	-0.2
18	3.20	10.24	24	3.00	9	9.6
20	5.20	27.04	25	4.00	16	20.8
16	1.20	1.44	22	1.00	1	1.2
22	7.20	51.84	20	-1.00	1	-7.20
∑X=148		$\sum x^2$=207.56	∑Y= 210		$\sum y^2$ =84	∑xy=-8

Covariance = ∑xy/n = -0.80

Variance of (X) = $\sum x^2/n$ =207.56/10 = 20.756 SD of X = 4.56

Variance (Y) =$\sum y^2/n$ = 84/10 = 8.40 SD of Y = 2.90

Coefficient of correlation = covariance /(SDx.SDy)=

-0.80/(4.56x2.90)= 0.065

Q.No.150: The following are the details of investments of Sachin Ganguly:

Equity shares of following companies	Amount of investment (Rs)	Expected return
A Ltd.	Rs.100000	10%
B Ltd	Rs.200000	11%
C Ltd.	Rs.300000	13%
D Ltd.	Rs.400000	14%
E Ltd.	Rs.100000	18%
F Ltd.	Rs.200000	20%

Find the expected return of Sachin's portfolio.

Answer

X	W	XW
10	10/130	0.76923
11	20/130	1.69231
13	30/130	3.00000
14	40/130	4.30769
18	10/130	1.38462
20	20/130	3.07692
Total	1	14.23

Expected return = 14.23/1 =14.23 %

Q.No.151: The equity shares of Devta Ltd. and Shree Ltd. have expected returns of 14% and 18% respectively while the standard deviations are 25 % and 35%. The correlation coefficient between expected returns from these two securities is 0.42. Find the expected return and variance of expected returns of a portfolio in which 20% of funds are invested in Devta Ltd. and balance in Shree Ltd.

Answer:

20 % in Devta and 80% in Shree

Portfolio Return = [14x.20] + [18x.80] = 17.20 %

Portfolio Variance :

$= (.20)^2.(0.25)^2 + (.80)^2.(0.35)^2 + 2(.20)(.80)(.42)(0.25)(0.35)$

$= 0.3044 = 30.44$ %

Q. No. 152: The correlation of coefficient between return from two securities, A and B, is −1. (i) Using the following information, suggest how an investor should invest Rs.500000 in these two securities so that risk is minimum.

	A	B
Expected return	17%	16%
S.D.	20%	30%

(ii) What will be your answer if coefficient correlation is 0.10 instead of −1?

Answer :

(i) $r = -1$ In this case, for minimum risk portfolio, the investment should be made in the reverse ratio of the SDS i.e. 30:20 i.e. 0.60 : 0.40

Invest Rs.300000 in A & Rs.200000 in B.

(ii)

$$W_A = \frac{(SD_B)^2 - r\,(SD_A)(SD_B)}{(SD_A)^2 + (SD_B)^2 - 2r(SD_A)(SD_B)}$$

$$= \frac{(30)^2 - (.10)(20)(30)}{(20)^2 + (30)^2 - 2(.10)(20)(30)}$$

$= 0.7119.$ $W_B = 0.2881$

Invest Rs.355950 in A & Rs.144050 in B

Q. No. 153: Find the Beta of portfolio of seven securities owned by Mr.X, details are as follows:

Security	Beta	Investment as % of total funds invested
A	1.50	10%
B	1.60	10%
C	0.90	20%
D	0.80	30%
E	0.70	10%
F	1.05	10%
G	1.25	10%

If risk free rate of return is 7% and expected return of market portfolio is 12%, find the expected return of the portfolio of Mr X.

Answer:

Portfolio Beta :

(1.50 x 0.10) + (1.60 x 0.10) + ……+ (1.25 x 0.10) = 1.03

Expected return of portfolio = 7 + 1.03(12−7) = 12.15 %

Q. No. 154: Find the expected return and standard deviation of a portfolio consisting of three securities A,B, and C, the details of which are given below. You may assume that 40% of funds are invested in A and 30% in each of B & C.

Securities	Expected return	Standard deviation
A	10%	.07
B	12%	.10
C	15%	.20

The correlation coefficient between returns of A&B is 0.50.The corresponding figures for A & C and B & C are 0.70 and 0.40 respectively.

Answer:
Expected return of portfolio = (10x0.40) + (12x0.30) + (15x0.30) = 12.10
Portfolio SD = 9.96%

Q. No. 155: SD of market returns is 30%, risk premium on the market portfolio is 10 %, RF is 6 %. Find the expected return and SD of a portfolio in which 0 % in Market and 100 % in RF, 20 % in market and 80 in RF, 40% in Market and 60 % in RF ………. 100% in market and 0% in RF.

Answer

Market	RF	Expected return	SD of portfolio
0	100	6	0
20	80	.20(16) + 0.80(6) = 8.00	.20(30) =6
40	60	.40(16)+ 0.60(6)= 10.00	.40(30) =12
60	40	.60(16)+ 0.40(6)= 12.00	.60(30) =18
80	20	.80(16)+ 0.20(6)= 14.00	.80(30) =24
100	0	16	30

Q. No.156: Your client is holding the following securities;

	Cost (Rs.)	Dividend(Rs.)	Market price (Rs.)	Beta
Equity shares				
Co.X	8000	800	8200	0.80
Co.Y	10000	800	10500	0.70
Co.Z	16000	800	22000	0.50
PSU Bonds	34000	3400	32300	1.00
Risk Free rate of return : 15%.				

Calculate (a) Expected rate of return in each, using the CAPM.
(b) Average return of the portfolio. **(May, 2003)**

Answer : Assumption: There are only these four securities available in the market.

$$RM = \frac{9000+11300+22800+35700}{8000+10000+16000+34000} - 1 = .1588$$

= 15.88%

Security	Expected return
X	15+ 0.80(15.88 -15) = 15.704
Y	15+ 0.70(15.88 -15) = 15.616
Z	15+ 0.50(15.88 -15) = 15.440
PSU Bonds	15+ 1.00(15.88-15) = 15.880

Average return of portfolio (Assumption : equal amount is invested in each security)

Return (R)	W	RW
15.704	0.25	3.9260
15.616	0.25	3.9040
15.440	0.25	3.8600
15.880	0.25	3.8875
		15.5775

$$\text{Average return} = \frac{\sum RW}{\sum W} = \frac{15.5775}{1} = 15.5775\ \%$$

Q. No. 157: A holds the following securities;

	Initial price (Rs.)	Dividend (Rs.)	Market price at end of year (Rs)	Beta
Equity shares				
Epsilon	25	2	50	0.80
Sigma	35	2	60	0.70
Omega	45	2	135	0.50
GOI Bonds	1000	140	1005	0.99
Risk Free rate of return : 14%.				

Calculate (a) Expected rate of return on his portfolio using the CAPM.

(b) Average return of the portfolio. **(May, 2008)**

Answer

Assumption: There are only these four securities available in the market.

$$RM = \frac{52+62+137+1145}{25+35+45+1000} - 1 = .2633$$

$= 26.33\%$

Security	Expected return
E.Epsilon Ltd	14+ 0.80(26.33 -14) =23.864
E. Sigma Ltd	14+ 0.70(26.33 -14) =22.631
E. Omega Ltd.	14+ 0.50(26.33 -14) =20.165
GOI Bonds	14+ 0.99(26.33-14) =26.2067

Average return of portfolio

(Assumption : equal amount is invested in each security)

Return (X)	W	XW
23.864	0.25	5.966
22.631	0.25	5.65775
20.165	0.25	5.04125
26.2067	0.25	6.551675
		23.216675

$$\text{Average return} = \frac{\sum XW}{\sum W} = \frac{23.216675}{1} = 23.22\ \%$$

Q. No. 158: Following is the data regarding six securities :

	A	B	C	D	E	F
Return	8	8	12	4	9	8
Risk	4	5	12	4	5	6

(i) Assuming three will have to be selected, state which ones will be picked

(ii) Assuming perfect correlation, show whether it is preferable to invest 75% in A and 25% in C or to invest 100% in E. **(Nov. 2002)**

Answer (i)

SET A:

(a) A, B and F have same return but A's SD is least. Hence, B and F are rejected.

(b) Now we are left with A, C, D and E. A and D have same SD but D's return is lower. Hence D is rejected. We are left with A, C and E.

SET B:

Security	Coefficient of variation
A	(4/8) x 100 = 50%
C	(12/12) x 100 = 100%
E	(5/9) x 100 = 55.56%

The securities may be selected in the following order:

(ii) A (ii) E (iii) C

Answer (ii)

Portfolio return = 9% SD = 6%

The investment may not be made in this portfolio as a better investment opportunity (investment in E) is available.

Q . No. 159 Following is the data regarding six securities :

	U	V	W	X	Y	Z
Return	10	10	15	5	11	10
Risk	5	6	13	5	6	7

(i) Which three will be selected ?

(ii) Assuming perfect correlation, show whether it is preferable to invest 80% in U and 20% in W or to invest 100% in Y. **(May. 2004)**

Answer (i)
SET A:
(a) U, V and Z have same return but U's SD is least. Hence, V and Z are rejected.
(b) Now we are left with U, W, X and Y. U and X have same SD but X's return is lower. Hence X is rejected. We are left with U, W and Y. These three securities may be selected.

Answer (ii)
Portfolio return =11%   SD = 6.60%
The investment may not be made in this portfolio as a better investment opportunity (investment in Y) is available.

Q. No.160: The results of four securities for a 5-years period are as follows: (RF 8%, RM 14%)

Securities	Average Return (%)	Beta
A	13	0.80
B	14	1.05
C	17	1.25
D	13	0.90

Which securities are overvalued? Undervalued?
Answer
AVERAGE RETURN IS LIKELY RETURN

SECURITY	LIKELY RETURN	EXPECTED RETURN	COMMENT
A	13%	8+ 0.80(14-8) =12.8	UNDERVALUED
B	14%	8+ 1.05(14-8)=14.30	OVERVALUED
C	17%	8+ 1.25(14-8)=15.50	UNDERVALUED
D	13%	8+ 0.90(14-8)=13.40	OVERVALUED

Q. No. 161: Mr Dravid invested 75% of his funds in security E and 25% in F, both securities have a Beta of 1.40. SD of market portfolio is 25%. Risk premium of market portfolio is 10%. Find expected risk premium from the portfolio of E and F.

Answer : Weighted average Beta of portfolio = 1.40
Expected risk premium of portfolio = 1.40 x 10 = 14%

Q. No. 162: XYZ Ltd has cash flow and until the surplus funds are utilized to meet the future capital expenditure, likely to happen after several months, are invested in a portfolio of short-term equity investments, details for which are given below:

Investment	No. of shares	Beta	Market price per share (Rs.)	Expected dividend yield (%)
I	60,000	1.16	4.29	19.50
II	80,000	2.28	2.92	24.00
III	1,00,000	0.90	2.17	17.50
IV	1.25,000	1.50	3.14	26.00

The current market return is 19% and risk free rate is 11%. Required to :

(i) calculate the risk of XYZ's short term investment portfolio relative to that of the market

(ii) whether XYZ should change the composition of its portfolio. **(Nov.'07)**

Answer

Assumption: Ex-dividend market price per share at the time of purchase of the shares = Expected ex-dividend market price at the time of expiry of the time horizon for which the investment is made.

Working note :

Investment	Amount of investment	Proportion of investment
I	60,000x4.29 =2,57,400	0.2339
II	80,000x2.92 =2,33,600	0.2123
III	1,00,000x2.17 =2,17,000	0.1972
IV	1,25,000x3.14 =3,92,500	0.3566
Total	11,00,500	1.00

(i) Risk of portfolio: Portfolio Beta

1.16(0,2339) + 2.28(0.2123) + (0.90)(0.1972) + 1.50(0.3566) = 1.47

The portfolio is 1.47 times riskier than the market portfolio.

(ii) Expected return

a. : 11 + 1.16(8) = 20.28

b. : 11 + 2.28(8) = 29.24

c. : 11 + 0.90(8) = 18.

d. : 11 + 1.50(8) = 23.00

e.

Investment	Expected return (risk based)	Expected return (Likely)	Comment
I	20.28	19.50	Overvalued
II	29.24	24.00	Overvalued
III	28.20	17.50	Overvalued
IV	23.00	26.00	Undervalued

The composition of the investment may be changed. Securities I, II and III should be replaced with some other good investments (including IV).

Q. No.163: A company has a choice of investments between several different equity oriented mutual funds. The company has an amount of Rs.1 Crore to invest. The details of the mutual funds are as follows:

Mutual fund	Beta
A	1.60
B	1.00
C	0.90
D	2.00
E	0.60

Required;

(i) If the company invests 20% of its investment in the first two mutual funds and an equal amount in the mutual funds C, D and E, what is the Beta of the portfolio?

(ii) If the company invests 15% of its investment in C, 15% in A, 10% in E and the balance in equal amount in the other two mutual funds, what is the Beta of the portfolio?

(iii) If the expected return of the market portfolio is 12% at a Beta factor of 1, what will be the portfolios' expected return in both the situations given above. **(May, 2008)**

Answer

(i) Assumption: the company invests 10% in each of A and B mutual funds.

Portfolio Beta = (0.10)(1.60 + 1.00) + (0.80/3)(0.90+ 2.00+ + 0.60) =1.1933

(ii) Portfolio Beta :

0.15(1.60+ 0.90) + 0.30(1.00+ 2.00) + 0.10(0.60) = 1.335

(iii) Assumption : RF = 10%

I case : Expected return = 10 + 1.1933 (2) = 12.39%

II case : Expected return = 10 + 1.335 (2) = 12.27%

Q. No.164 Consider the following information on two stock, A and B :

Year	Return on A (%)	Return on B (%)
2006	10	12
2007	16	18

You are required to determine:

(i) The expected return on a portfolio containing A and B in the proportion of 40% and 60%

(ii) The SD of return from each of the two stocks
(iii) The covariance of returns from the two stocks
(iv) Correlation coefficient between the returns of the two stocks
(v) The risk of a portfolio containing A and B in the proportion of 40% and 60%. **(Nov. 2008 SFM)**

Answer : (i) Expected return of A = (10+ 16) / 2 = 13%
Expected return of B = (12+ 18) / 2 = 15%
Expected return of Portfolio = 13 x 0.40 + 15 x 0.60 = 14.20%

(ii) Calculation of SD of A :

X	x	x^2
10	-3	9
16	+3	9
		$\sum x^2$ =18

SD of A = $\sqrt{(18/2)}$ = 3

Calculation of SD of B :

Y	y	y^2
12	-3	9
18	+3	9
		$\sum x^2$ =18

SD of B = $\sqrt{(18/2)}$ = 3

(iii) Covariance of Returns from two stocks

X	x	Y	y	xy
10	-3	12	-3	9
16	+3	18	+3	9
	$\sum x$ =0		$\sum y$ = 0	$\sum xy$ =18

Covariance = $\sum xy$ / n = 18/2 = 9

(i) Coefficient of Correlation = Covariance / (SD_A x SD_B)
= 9 / (3 x 3) = 1

(ii) Portfolio Risk =

$$\sqrt{(W_1)^2.(SD_1)^2 + (W_2)^2.\ (SD_2)^2 + 2(W_1)(W_2)(r_{12})(SD_1)(SD_2)}$$

$$= \sqrt{(0.40)^2.\ (3)^2 + (0.60)^2.\ (3)^2 + 2(0.40)(0.60)(1)(3)(3)} = 3$$

Q. No.165 An investor holds two stock A and B. AN analyst prepared ex-ante[19] probability distribution for the possible economic scenarios and conditional returns for the two stocks and the market Index as follows:

Economic scenario	Probability	Conditional returns%		
		A	B	Market
Growth	0.40	25	20	18
Stagnation	0.30	10	15	13
Recession	0.30	-5	-8	-3

The risk free rate during the next year is expected to be 11%. Determine whether the investor should liquidate his holdings in A & B or on the contrary make fresh investment in them. CAPM assumptions are holding true. **(NOV. 2009 SFM)**

Answer:

Expected return of A : 25 x 0.40 + 10 x 0.30 -5 x 0.30 = 11.50%
Expected return of B : 20 x 0.40 + 15 x 0.30 -8 x 0.30 = 10.10%
Expected return market : 18 x 0.40 + 13 x 0.30 -3 x 0.30 = 10.20%

Calculation of Market variance : (Let the market return = Z)

Returns (Z)	z	p	Pz^2
18	7.80	0.40	24.336
13	2.80	0.30	2.352
-3	-13.20	0.30	52.272
			$\sum pz^2 = 78.96$

Market variance = 78.96

Covariance between A and market : (Let the return from A = X)

x	z	p	pxz
13.50	7.80	0.40	42.12
-1.50	2.80	0.30	-1.26

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[19] The term ***ex-ante*** is a Latin word meaning "*before the event*". Ex-ante is used most commonly in the commercial world, where results of a particular action, or series of actions, are forecast in advance.

-16.50	-13.20	0.30	65.34
			∑pxz = 106.20

Covariance between A and market = 106.20

Covariance between B and market : (Let the return from B = Y)

y	z	p	pyz
9.90	7.80	0.40	30.888
4.90	2.80	0.30	4.116
-18.10	-13.20	0.30	71.676
			∑pyz = 106.68

Covariance between B and market = 106.68

Beta = Covariance between security and market / market variance
Beta of A = 106.20/78.96 = 1.345
Beta of B = 106.68/78.96 = 1.351

Expected return : RF + Beta (RM-RF)
Expected return of A = 11 + 1.345(10.20-11) = 9.924%
Expected return of B = 11 + 1.351(10.20-11) = 9.920%
As the actual returns of both the securities are higher than their expected returns, both the securities are undervalued i.e. more amount of investment should be made in these securities.

Q. No.166 The returns on stock A and market portfolio for a period of 6 years are as follows:

Year	Return on A (%)	Return on market portfolio (%)
1	12	8
2	15	12
3	11	11
4	2	-4
5	10	9.5
6	-12	-2

You are required to determine:
(i) Characteristic line for stock A
(ii) The systematic and unsystematic risk of stock A. (8 Marks)(**June 2009**)

Answer

(i) Calculation of Beta and SD of A

Return on A % (X)	Return on market Portfolio (Y)	x	y	x^2	y^2	xy
12	8	5.67	2.25	32.15	5.0625	12.7575
15	12	8.67	6.25	75.17	39.0625	54.1875
11	11	4.67	5.25	21.81	27.5625	24.5175
2	-4	-4.33	-9.75	18.75	95.0625	42.2175
10	9.50	3.67	3.75	13.47	14.0625	13.7625
-12	-2	-18.33	-7.75	335.99	60.0625	142.0575
$\sum X = 38$	$\sum Y = 34.50$			$\sum x^2 =$ 497.34	$\sum y^2 =$ 240.875	$\sum xy =$ 89.50 2

Mean return of A = 38/6 = 6.33   Mean Return of market: 34.50/6 = 5.75

Variance of A = 497.34 / 6 = 82.89

Variance of Market = 240.875 / 6 = 40.15

Covariance = 289.50 / 6 = 48.25

Beta = 48.45 / 40.15 = 1.21

Alpha = mean return of security – Beta x RM

= 6.33 – 1.21 x 5.75 = – 0.6275

Characteristic Line = Alpha + Beta x RM

= – 0.6275 + 1.21 x RM

(ii) Total risk of A = 82.89

Systematic risk of A = $Beta^2$ x market variance = 58.78

Unsystematic risk of A = Total risk of A – systematic risk of A

= 82.89 – 58.78 = 24.11

Q. No.167 A stock costing Rs.120 pays no dividend. The possible prices of the stock at the end of the year are given below with respective probabilities:

Price	115	120	125	130	135	140
Probability	0.10	0.10	0.20	0.30	0.20	0.10

Calculate the expected return. Calculate the SD of the returns.

Answer

Price	Wealth ratio	Return (%) (X)	p	pX	x	px^2
115	115/120 = 0.9583	- 4.17	0.1	-0.42	11.25	12.66
120	120 /120 = 1	0	0.1	0	- 7.08	5.02
125	125/120 = 1.0417	4.17	0.2	0.83	-2.91	1.69
130	130/120 = 1.0833	8.333	0.3	2.50	1.253	0.47
135	135/120 = 1.125	12.50	0.2	2.50	5.42	5.88
140	140/120 = 1.1666	16.67	0.1	1.67	9.59	9.20
				$\sum pX$ = 7.08		$\sum px^2$ = 34.92

Expected return = $\sum pX/\sum p$ = 7.08/1 = 7.08

Standard Deviation =$\sqrt{(\sum px^2 / \sum p)}$ =$\sqrt{(\sum 34.92 / 1)}$ = 5.91

Q. No.168 ABC Ltd has been maintaining a growth rate of 10% in dividends. The company has paid dividend of Rs.3 per share. The rate of return on market portfolio is 12% and the risk free rate of return in the market has been observed as 8%. The Beta co-efficient of the company's share is 1.5.You are required to calculate the expected rate of return on the company's share as per CAPM model and equilibrium price per share by dividend growth model.**(Nov.2008)**

Answer :

Ke = Expected rate of return on equity share = RF + Beta(RM - RF)
= 8 + 1.50(12-8) = 14%

$$\text{Equilibrium price per share} = \frac{3(1.10)}{0.14 - 0.10} = 82.50$$

Q. No.169 Mr A is interested to invest Rs.1,00,000 in the securities market. He selected two securities B and D for this purpose. The risk return profile of these securities are as follows:

Secuirty	Risk (SD)	Expected Return
B	10%	12%
D	18%	20%

The co-efficient of correlation between B and D is 0.15.

You are required to calculate the portfolio risk and portfolio return of the following portfolios of B and D to be considered by A for his investment:

(i) 100% investment in B only
(ii) 50% of the funds invested in B and D both
(iii) 75% of the fund in B and rest 25% in D
(iv) 25% of the fund I n B and rest 75% in D
(v) 100% investment in D only

Also indicate that which portfolio is the best for him from risk as well return point of view? **(Nov. 2008)**

Answer

(i) 100% investment in B only

Return : 12% Risk = 10

(ii) 50% of the funds invested in B and D both

Return = 12(0.50) + 20 (0.50) = 16

Portfolio variance = $(W_{B.}SD_{B})^2 + (W_{D.}SD_{D})^2 + 2W_{B.}W_{D.}r_{BD.}SD_{B.}SD_{D}$

$=(0.50x0.10)^2+(0.50x0.18)^2+ 2x0.50x0.50x0.15x0.10xx0.18$

= 0.01194

Portfolio (Risk) : SD = 0.10.93 = 10.93%

(iii) 75% of the funds invested in B and 25% in D

Return = 12(0.75) + 20 (0.25) = 14

Portfolio variance = $(W_{B.}SD_{B})^2 + (W_{D.}SD_{D})^2 + 2W_{B.}W_{D.}r_{BD.}SD_{B.}SD_{D}$

$=(0.75x0.10)^2+(0.25x0.18)^2+ 2x0.75x0.25x0.15x0.10xx0.18$

= 0.00866

Portfolio (Risk) : SD = 0.09307 = 9.307%

(iv) 25% of the funds invested in B and 75% in D

Return = 12(0.25) + 20 (0.75) = 18

Portfolio variance = $(W_{B.}SD_B)^2$ + $(W_{D.}SD_D)^2$ + $2W_{B.}W_{D.}r_{BD.}SD_{B.}SD_D$

= $(0.25x0.10)^2$+ $(0.75x0.18)^2$+ 2x0.25x0.75x0.15x0.10xx0.18

= 0.01986

Portfolio (Risk) : SD = 0.1409 = 14.09%

(v) 100% investment in D only : Return : 20% Risk = 18

Q. No.170 A firm paid a dividend of Rs.2 last year.. The estimated growth rate of the dividend is 5%.Determine the estimated market price of the share if the estimated growth rate (i) rises to 8% and (ii0 falls to 3%. Also find the present price of the share assuming Ke = 15.50% **(NOV. 2009 SFM)**

Answer

Market price of share :

$$P = \frac{D_1}{K_e - g}$$

Current market price = 2(1.05) /(0.155-0.05) = 20

Market price if growth rate is 8% = 2(1.08) /(0.155-0.08) = 28.80

Market price if growth rate is 3% = 2(1.03) /(0.155-0.03) = 16.48

THEORETICAL ASPECTS

Q. No. 171 : Write a short note on systematic and unsystematic risk in connection with the portfolio investment. **(May, 1999)**

Distinguish between Systematic and Unsystematic risk. **(Nov. 2004)**

Discuss various kinds of systematic and unsystematic risks. **(Nov. 2006)**

Answer : Unsystematic Risk

It is also known as micro level risk. It is concerned with the company or industry. Strike, wrong decisions by the management, change in management, increase in input costs (without increase in sale price), change in government policy regarding particular type of companies or products, emerging of substitutes of the company's product(s), cancellation of export order, key-person leaving the company, fire, embezzlement by employees, unexpected tax demand, major problem in the plant, etc. The incidence of such risks can be reduced through effective portfolio selections. The two serious unsystematic risks are:

(i) Business risk: Business risk is the possibility of adverse change in EBIT. Examples are: Reduction in demand for company's products, increase in costs of inputs, change in import-export policy concerning the company, Labour strike, some key-person's leaving the company, cancellation of large sized export order etc.

(ii) Financial risk: It is the possibility of bankruptcy. It arises because of dependence on borrowed funds and that to it high interest rates.

(iii) Default risk: The major customer of the company may go bankrupt.

Systematic Risk

It is known as macro level risk. It is concerned with the economy as a whole. The factors causing this type of risk affect all the investments in a similar fashion (and not in a similar degree). Examples are : failure of monsoon, change in government, change in credit policy, recession, war, change in tax policy, etc. Every portfolio has to bear this risk. The two most serious systematic risks are :

(i) Interest rate risk : increase in interest rates generally have adverse effects on the financial position and earnings of the companies.

(ii) Inflation risk : inflation disturbs business plans of the most of the organizations. Input costs m ay go up, all the increase in input costs may not been passed to the customers.

(iii) Political risk : This risk involves (a) change in government policies and political instability.

Q. No. 172: " Higher the return, higher will be the risk". In this context discuss the various risks associated with portfolio planning. **(Nov. 1996)**

Answer : First deserve and then desire is an old maxim. The wisdom of this maxim is that higher return should be expected only by those who are willing to bear higher risk. If an investor is not willing to go for higher risk, he should invest in risk-free securities (say, for example Government securities) and naturally he should not expect the higher return. There are two parts of return from investment (i) Risk-free return, and (ii) risk premium. It is the risk premium that enhances the return from the investment. Risk premium is a function of risk. Risk premium changes in direct proportion of risk (return does not change in direct proportion of risk). Hence, higher return should be expected only by those who bear higher degree of risk.

The statement that Higher the return, higher will be risk needs to be amended. The word will should be substituted by the word may. Higher risk may result in one of the following three mutually exclusive cases : (i) Higher return, (ii) Lower return, (iii) Negative return. Taking higher return is no guarantee of higher return. Everyone wants higher return; if taking higher risk is guarantee of higher return, no one will go for lower risk.

Higher risk may not result in higher return because of the following risks associated with the investments:

Unsystematic Risk : Refer to answer to Question 1

Systematic Risk : Refer to answer to Question 1

Q.No.173: Briefly explain Capital asset Pricing Model. **(Nov 1997; May 2003)**

(Nov. 2009) Assumptions of CAPM. **(May, 2006) (N0v. 2008)**

Answer:

For appreciating the CAPM, we have to understand different types of risks on the investments.

The required rate of return on the investments depends on the riskiness of the investments. Lesser the risk, lesser the required rate of return and vice-versa. The risks on the investments can be decomposed in two parts:

(i) Systematic Risk
(ii) Unsystematic risk

Systematic risk refers to variability in return on investment due to market factors that affect all investments in a similar fashion.
Theme of the CAPM is that the investors need to be compensated for (i) Time value of money and (ii) Risk they have taken.

The required rate of return can be divided in two parts:

(i) Compensation for time value of money. It is represented by Risk free rate of return

(ii) Compensation for taking the risk i.e. Risk premium.

- The required risk premium on account of systematic risk can be estimated with the help of Beta.

BETA

Beta is an indicator of an investment's systematic risk. It measures systematic risk associated with an investment in relation to total risk associated with market portfolio. Suppose the beta value of a particular security is 1.20, it means that if return of market portfolio varies by one per cent, the return from that security is likely to vary by 1.20 per cent. Therefore, this security is riskier than the market because we expect its return to fluctuate more than the market on a percentage basis. This beta measures the riskiness of individual security relative to market portfolio. It is a ratio of "its covariance with the market" to "the variance of market as a whole". A security with beta greater than one is called as aggressive security; with beta less than one is called as defensive security and with beta equal to one is called as neutral security.

CAPM explains the required return (i.e. the minimum rate of return which induces the investors to select a particular investment) in the form of the following equation:

K = RF + RP
K = Required rate of return
RF = Risk free rate of return
RP = Risk premium

Risk premium is additional return expected by the investor for bearing the additional Systematic risk associated with a particular investment. It is calculated as:

Beta X (RM-RF)

Where RM is expected return on market portfolio. The Beta value that we take here should be corresponding with security. For example, if we have to find the required rate of return of a share, we should consider the Beta of that share; if we have to find the required rate of return of a debenture, we should consider the Beta of that debenture; if we have to find the required rate of return of a portfolio, we should consider the Beta of that portfolio.

Suppose beta of a security is 1.21

RF = 7 per cent, RM = 13 per cent
K = 7 + 1.21 (13 - 7) = 14.26 per cent

- Investor will require a return of 14.26 per cent return from this investment.
- He can get 7 per cent return without taking any risk.
- Market portfolio offers him extra 6 per cent return where risk is lesser as compared to risk from this security.
- Risk from this security is 1.21 times as compared to risk from market portfolio. Hence premium is 6 x 1.21 = 7.26 per cent.

- Thus required rate of return is equal to risk free return + risk premium.

The attraction of the CAPM is its powerfully simple logic and intuitively pleasing predictions about how to measure risk and about the relation between expected return and risk.

CAPM is based on the following assumptions :

- There are no taxes or transaction costs.
- Investors always desire more return to less, and they are risk averse;
- All investors have identical investment time horizons.
- All investors have identical opinions about expected returns and volatilities
- There are no restrictions on the borrowing and lending of money at the risk-free rate of interest.
- All investments are traded in the market, the assets are infinitely devisable, and there are no restrictions on short selling.
- The market is perfectly efficient. That is, every investor receives and understands the same information and processes it accurately
- All investors have expectations. They know that higher returns can be earned only by taking enhanced risk. They are rational and know their risk tolerance capacity.
- There are no arbitrage opportunities.
- Returns are distributed normally.
- No inflation and no change in the level of interest rate exist.

Q. No. 1744: Explain briefly the two basic principles of effective portfolio management. **(May 1996; Nov. 1999)**

Answer :

The two basic principles of effective portfolio management are :

(i) Invest on the basis of fundamentals of the security.

(ii) Review and update the portfolio regularly.

The object of the portfolio management is to provide maximum return on the investments by taking only optimum risk. To achieve these objectives, the portfolio manager should invest in diversified securities and see that the coefficient of correlation between these securities is as less as possible (only then the portfolio will be able to reduce the risk). This is the foundation of portfolio management. The portfolio manager should follow the above-mentioned principles to further strengthen his targets of higher returns and optimum risk.

The first principle suggests that investment should be made only in those securities which are fundamentally strong. The strength of a security depends upon three strengths: (a) strength of the company, (b) strength of the industry, and (c) strength of the economy. The strength of the company depends upon various factors like (i) intelligent, dedicated and motivated human resources, (ii) management having positive values and vision, (iii) policy regarding encouraging R&D, (vi) integrity of promoters, and (v) long range planning for profits.

The second principle suggests that the portfolio should be reviewed continuously and if need be, revised immediately. The Fundamentals of the company, industry and economy keep on changing. Accordingly, the portfolio should be revised according to emerging situations. For example, in case of monsoon failure, investments should move from fertilizer companies to irrigation companies, in case some sick-minded person takes over as CEO of the company, perhaps desired step will be to disinvest the securities of the company, in case cheaper substitutes have emerged for any industry's product, better move to some other industry, etc.

Two more points regarding the second principle

(i) Sometimes, after making the investment in some securities, portfolio manager realizes that his decision of investing in that security is wrong, he should not wait for happening of some event which will make his decision as a right one (if there is some loss on that investment, he should not even wait for breakeven); rather he should move immediately liquidate his position in that security.

(ii) Do not bother much about transaction cost related to reshuffling of the portfolio, consideration of such small costs generally result in heavy losses or foregone opportunities of earning profit.

Q. No.175: Write a short note on the objectives of portfolio management. **(Nov. 1998)**

Answer:

OBJECTIVES OF PORTFOLIO MANAGEMENT

There are seven objectives of portfolio management:

Return

Portfolio management is technique of investing in securities. The ultimate object of investment in the securities is return. Hence, the first objective of portfolio management is getting higher return.

Capital Growth

Some investors do not need regular returns. Their object of portfolio management is that not only their current wealth is invested in the securities; they also want a channel where their future incomes will also be invested.

Liquidity

Some investors prefer that the portfolio should be such that whenever they need their money, they may get the same.

Availability of Money at Pre-decided Time

Some persons invest their money to use it at pre-decided time, say education of children, etc. Their objective of portfolio planning would be that they get their money at that time.

Favourable Tax Treatment

Sometimes, some portfolio planning is done to obtain some tax savings.

Maintaining the Purchasing Power

Inflation eats the value of money, i.e., purchasing power. Hence, one object of the portfolio is that it must ensure maintaining the purchasing power of the investor intact besides providing the return.

Risk Reduction through Diversification

It is the perhaps most important object of the portfolio management. All other objectives (mentioned above) can be achieved even without portfolio, i.e., through investment in a single security, but reduction (without sacrificing the return) is possible only through portfolio.

Single most important objective of the portfolio management is risk reduction through diversification.

Q. No. 176: Write note on factors affecting investment decisions in portfolio management. (**May, 2000**)

Answer:

Factors affecting investment decisions in portfolio/asset allocation in portfolio are as follows:

Risk Tolerance

RISK refers to the volatility of portfolio's value. The amount of risk the investor is willing to take on is an extremely important factor. While some people do become more risk averse as they get older; a conservative investor remains risk averse over his life-cycle. An aggressive investor generally dares to take risk throughout his life. If an investor is risk averse and he takes too much risk, he usually panic when confronted with unexpected losses and abandon their investment plans mid-stream and suffers huge losses.

Return Needs

This refers to whether the investor needs to emphasize growth or income.

Investment Time Horizon

The time horizon starts when the investment portfolio is implemented and ends when the investor will need to take the money out. The length of time you will be investing is important because it can directly affect your ability to reduce risk. Longer time horizons allow you to take on greater risks with a greater total return potential. If the time horizon is short, the investor has greater liquidity needs some attractive opportunities of earning higher return has to be sacrificed and the result is reduced in return.

Tax Exposure

Investors in higher tax brackets prefer such investments where the return is tax exempt, others will have no such preference.

Q. No.177: (i) What sort of investor normally views the variance (or Standard Deviation) of an individual security's return as security's proper measure of risk?
(ii) What sort of investor rationally views the beta of a security as the security's proper measure of risk? In answering the question, explain the concept of beta.
(May, 2004)

Answer : (i) Investor with long-term time horizon (investing from long term point of view) view SD as the proper measure of security's risk. SD is a measure of total risk and if the investment is from long term point of view total risk should be considered.
Longer the period, larger the risk - as in long run fundamentals of the economy as well as company may change. All these changes are reflected in SD of past returns of security (the implied assumption is that the history repeats itself). **(ii)** Investor with short run time horizon view beta as the proper measure of risk. Beta measures systematic risk of the security. Any bad news (say no-trust motion against government, slightest possibility of war, death or serious illness of some key person of the economy) may upset the market and result is adverse impact on the price of the security. If beta of the security is high, even slight adverse factor resulting in slight adverse impact on the market may have substantial adverse impact on price of the security.

Concept of Beta: Beta is an indicator of an investment's systematic risk. It measures systematic risk associated with an investment in relation to total risk associated with market portfolio. Suppose the beta value of a particular security is 1.20, it means that if return of market portfolio varies by one per cent, the return from that security is likely to vary by 1.20 per cent. Therefore, this security is riskier than the market because we expect its return to fluctuate more than the market on a percentage basis. This beta measures the riskiness of individual security relative to market portfolio. It is a ratio of "its covariance with the market" to "the variance of market as a whole". A security with beta greater than one is called as aggressive security; with beta less than one is called as defensive security and with beta equal to one is called as neutral security.

Q. No.178: Explain Equity Style Management.

Answer
Equity investment, a complex process, requires a lot of expertise, experience and time. Small investors do not have time, skill or access to the information to assess available investment opportunities and manage their money most effectively. They do not have sufficient money to invest for reducing the risk through diversification. That's why generally such investors outsource the investment function to outside agencies like portfolio managers, mutual funds, pension funds etc. These agencies pool the investors' money and make collective investments on behalf of their clients/customers. The investment is managed by professional fund managers ensuring higher return, lesser risk and adequately liquidity. This has led to reduced direct equity investment by individuals and a

corresponding increase in the institutional investing. While holding equities through money management institutions has made it possible for individual investors to reap diversification benefits and to benefit from expertise of portfolio managers, it is not been without cost. Investors have to bear the cost for this approach.

Equity Style Management is the new frontier in institutional investment management. Professional portfolio managers generally do not have a choice about applying the general investment philosophy to govern the portfolios they manage. They are constrained either by

- the client's guidelines and risk tolerance, or
- Self- stated investment objectives (This is generally applicable in case of mutual funds schemes. Such schemes define and confine the investment strategy)

The concept of the equity style management requires the professional fund managers to achieve the higher returns for their clients while operating as per the pre-determined general investment philosophy i.e. "the guide lines of the client" / "self- stated investment objectives". The term 'style' here refers to equity exposure that the portfolio manager should take to lead to / exceed the expected returns. A group of securities is said to belong to a similar 'style' when there is a high degree of co-movement in their prices. It has been observed that there are categories of equity shares that have similar characteristics and performance patterns. Also, the returns of these equity shares categories performed differently than other categories of the equity shares. The returns of shares of the same category are highly high degree of positive correlation.

Style is referred as an investment scheme that groups stocks based on common characteristics. There are different types of the styles that the fund managers may implement. For example, growth stocks, value stocks, small cap stocks, mid capitalization stocks, large caps etc. Each of these styles has its distinctive features including distinctive risk characteristics. Equity style management aims to identify and describe the characteristics of an investment portfolio. Style classes are not static. A large cap value or small cap growth stock doesn't stay that way forever. PE ratios and market caps change every day. So, today's small cap value stock may become tomorrow's large cap growth stock, and vice versa.

The investors use style to (i) understand what types of investments their fund managers are buying for them (ii) monitor style consistency and (iii) control risk. Ongoing assessments of style are important because the investors need to ensure that managers do what is assigned. For this purpose the performance of the fund manager is compared with some bench mark. For example, the return of large cap style can be compared with the return on Nifty. Also the performance of a fund manager managing a particular style can be compared with that of other fund managers managing the similar types of equity styles. The performance assessment is done on the basis of risk adjusted returns.

There are two approaches regarding managing the equity styles: (i) Active approach and (ii) Passive approach.

- **Active approach**: Active investing approach aims at beating the market. The philosophy of the active investing is share markets are not fully efficient. There are situations when the shares are mispriced and there are opportunities to profit for the smart investors.
  incur greater transactions and advisory costs.

Passive approach: Passive investing approach believes that it is not possible to beat the market. The philosophy of the active investing is share markets are fully efficient. One can earn only normal returns (risk based returns) from the share market. The shares are not mispriced at any time. Hence, neither the share selection nor the timing results in super-normal return. Passive managers invest in broad sectors of the market and expect the average normal returns.

Different styles:
Though the equity shares can be classified into styles in different ways, the most popular styles are 'growth' and 'value'. In other words, we can say that growth and value are styles of investing in equity shares.

Growth shares are associated with high-quality; successful companies whose earnings are expected to continue growing at an above-average rate relative to the market. Growth shares generally have
the following characteristics:
(i) high price-to-earnings (P/E) ratios
(ii) high price-to-book ratios.
(iii) show sound growth rates and potential for future growth.
(iv) expected to give larger capital appreciation in future
(v) represent a relatively safe investment

Value shares are those, which the market has overlooked and as a result their price is low (relative what it is worth) and about to increase when the necessary market corrections occur. Value shares generally have the following characteristics :
(i) low price-to-earnings (P/E) ratios
(ii) low price-to-book ratios.
(iii) value stocks generally have good fundamentals, but they may have fallen out of favour in the market.
(iv) represent a more risky investment

Neither approach guarantees to provide positive returns (not to talk of supernormal return); both carry investment risk.

Assets Allocation :

Asset allocation accounts for a large part of the variability in the return on a portfolio. Asset allocation is generally defined as the allocation of an investor's investible funds among a number of "major" asset classes. This allocation aims at value creation but it is constrained by the risk-aversion of the investor. If the investor is not risk-averse, the major portion of the investible funds may be allocated to equity; if the investor is risk-averse, the major portion of the investible funds may be allocated to fixed income securities.

Style decision :

Asset allocation involves a risk decision. This decision is followed by 'Style' decision. The style decision is guided by both risk and return. The investor has to decide whether he wants to go for large cap growth shares or mid cap value shares etc. The portfolio manager may advise the investor on this matter but the decision is to be taken by the investor.

Investment Philosophies:

There are two investment philosophies followed by the portfolio managers:
(i) Top down philosophy. and
(ii) Bottom up philosophy.

Top down philosophy follows the following investment process (a) First consider the macro-factors i.e. the state of economy; invest in the economy that is strong and growing (b) then, consider the industry; invest in the industry which is expected to outperform other industries (c) finally, consider the company; invest in the company which is expected to be best in the industry.

Bottom up philosophy gives maximum weight to the 'company' i.e. a bottom-up investor considers the financial health, products, supply and demand, and other aspects of a company's performance over a given period of time. Using this approach the portfolio manager pay less attention to the economy as a whole, or to the prospects of the industry a company is in.

Performance Evaluation:

A portfolio is judged against a benchmark by comparing a series of returns from the portfolio with the corresponding returns for the benchmark. Suppose the bench mark provides a risk premium of 10%, its standard deviation of the returns from this bench mark is 5. Now further suppose that the standard deviation of returns from the portfolio under assessment is 6. The portfolio should provide minimum risk premium of 12%. If the risk free rate of return is 10%, the portfolio manager is required to provide minimum 22% return.

Q. No.179 : Write a note o n Arbitrage Operations. **(Nov. 2008)**

Answer Arbitrage is the process of taking advantage of the price differential. Arbitrage actions by different arbitrageurs tend to remove the price differential. Arbitrage is possible in the following three cases :

(i) The same asset quotes at different prices in different markets. For example, prices of shares of a particular company in BSE and NSE

(ii) There are different expected returns for two or more similar assets (i.e. the assets having the same risk). For example, suppose there are two assets having the same risk, theoretically, their expected returns must be same. If their returns are different, the arbitrageur will sell the assets having lower return and buy the assets having the higher returns and have some profit.

(iii) The realization from an asset on a future date is certain. If today its price is not equal to the present value of the amount to be realized, the present value should be calculated using risk free rate.

Example ; APPENDIX A

	SD	SD%	Variance	Variance%
(a)	2	?	?	?
(b)	0.07	?	?	?
(c)	?	7	?	?
(d)	?	40	?	?
(e)	?	?	16	?
(f)	?	?	9	?
(g)	?	?	?	36
(h)	?	?	?	64
(i)	?	?	?	100

Answer:

	SD	SD%	Variance	Variance%
(a)	2	200	4	400
(b)	0.07	7	0.0049	0.49
(c)	0.07	7	0.0049	0.49
(d)	0.40	40	0.16	16
(e)	4	400	16	1600
(f)	3	300	9	900
(g)	0.60	6	0.36	36
(h)	0.80	80	0.64	64
(i)	1	100	1	100

Example : Find the (i) portfolio Variance (ii) portfolio SD, using the following data.

Secruity A SD = 36%

Security B SD = 64%

r between returns from the two securities = −1

Equal amount of investment is made in each of the two securities.

Answer

Portfolio Variance =

$$(0.50)^2(0.36)^2 + (0,50)^2(0.64)^2 + 2(0.50)(0.50)(-1)(0.36)(0.64) = 0.0196$$

= 1.96%

Alternative way:

Portfolio Variance =

$$[(0.50)^2(36)^2 + (0,50)^2(64)^2 + 2(0.50)(0.50)(-1)(36)(64)] / 100 = 1.96\%$$

Portfolio SD:

$$\sqrt{(0.50)^2(0.36)^2 + (0,50)^2(0.64)^2 + 2(0.50)(0.50)(-1)(0.36)(0.64)} = 0.14$$

Alternative way:

Portfolio SD =

$$\sqrt{[(0.50)^2(36)^2 + (0,50)^2(64)^2 + 2(0.50)(0.50)(-1)(36)(64)]} = 14\%$$

APPENDIX B

DERIVATION FOR MINIMUM VARIANCE FORMULA

Let's invest w_1 in one and $(1- w_1)$ in other security. Let portfolio variance=y

$$Y = (w_1)^2 .(SD_1)^2 + (1- w_1)^2.(SD_2)^2 + 2(w_1)(1- w_1).r_{12}.(SD_1) (SD_2)$$

$$=(w_1)^2 .(SD_1)^2 + [1+(w_1)^2 -2 w_1] .(SD_2)^2 + 2(w_1)(1- w_1).r_{12}.(SD_1) (SD_2)$$

$$= (w_1)^2 .(SD_1)^2 + (SD_2)^2 + (w_1)^2.(SD_2)^2 -2 w_1. (SD_2)^2$$
$$+ 2(w_1).r_{12}.(SD_1) (SD_2) -2(w_1)^2.r_{12}.(SD_1) (SD_2)$$

$$dy/w_1 = 2(w_1) .(SD_1)^2 + 0 + 2(w_1).(SD_2)^2 -2. (SD_2)^2$$
$$+ 2.r_{12}.(SD_1) (SD_2) -4(w_1).r_{12}.(SD_1) (SD_2)$$

As per minima concept of differential Calculus, we get minimum value of y by putting $dy/w_1 =0$

$$2(w_1) .(SD_1)^2 + 2(w_1).(SD_2)^2 -2. (SD_2)^2 + 2.r_{12}.(SD_1) (SD_2)$$
$$-4(w_1).r_{12}.(SD_1) (SD_2) = 0$$

$$2(w_1) .(SD_1)^2 + 2(w_1).(SD_2)^2 -4(w_1).r_{12}.(SD_1) (SD_2)$$
$$= 2. (SD_2)^2 - 2.r_{12}.(SD_1) (SD_2)$$

$$(w_1) .(SD_1)^2 + (w_1).(SD_2)^2 -2(w_1).r_{12}.(SD_1) (SD_2)$$
$$= (SD_2)^2 - r_{12}.(SD_1) (SD_2)$$

$$w_1[(SD_1)^2 + (SD_2)^2 -2.r_{12}.(SD_1) (SD_2)] = (SD_2)^2 - r_{12}.(SD_1) (SD_2)$$

$$w_1 = \frac{(SD_2)^2 - r_{12}.(SD_1) (SD_2)}{(SD_1)^2 + (SD_2)^2 -2.r_{12}.(SD_1) (SD_2)}$$